Problem of the Week-3: Limit of a Sequence

Let a_n be a sequence defined by

$$a_1 = 1, a_2 = 2 \text{ and } a_{n+2} = \frac{a_{n+1} + a_n}{2} \text{ for all } n \geq 1.$$

Show that a_n converges and find $\lim_{n\to\infty} a_n$

As always, show your work, fully explain and justify your answer.