Problem of the Week-6: Continued Radicals

For any non-negative real number m, consider the expression

$$\sqrt{m+\sqrt{m+\sqrt{m+\ldots}}}$$

1. Show that this expression converges for any non-negative real number m. (Hint: Think of this as a limit of a recursively-defined sequence, and show that the sequence is increasing and bounded from above (by $1+\sqrt{m+1}$))

2. Find an expression for this infinite continued radical expression in terms of m that does not involve a continued radical. Then determine all positive integers m so that this expression turns out to be an integer.

Note: If you can only solve the second part still submit your solution. If I only receive correct answers to the second part, the winner will be chosen among those.

Posting Date 11/2/08. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 11/14/08.