Problem 6: Migrating Birds

Power (P) is the rate at which energy (E) is consumed per unit time. Ornithologists have found that the power consumed by a certain pigeon flying at velocity v m/s is described well by the function $P(v) = 17v^{-1} + 10^{-3}v^3$ J/s. Assume that the pigeon can store 5×10^4 J of usable energy as body fat.

1. Find the velocity that minimizes the power consumption.

2. Show that a pigeon flying at velocity v and using all of its stored energy can fly a total distance $D(v) = (5 \times 10^4)v/P(v)$.

3. Migrating birds are smart enough to fly at the velocity that maximizes distance traveled rather than minimizes power consumption. Show that the velocity v_{dmax} that maximizes the total distance satisfies $P'(v) = P(v)/v$.

4. Find v_{dmax} and the maximum total distance that the bird can fly.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.