Problem of the Week-7: Signed Binary Representations of Integers

A *signed binary representation* of a positive integer k given by $k = \sum_{i=0}^{t} k_i 2^i$ where each k_i is 0, 1 or -1. We also use the vector $(k_t, \ldots, k_0)$ to denote the representation where the right-most digit is the coefficient of the lowest order term.

1. Show that a signed binary representation of an integer is not unique.

   A signed representation $(k_t, \ldots, k_0)$ of k is said to be in *non-adjacent form* if no two consecutive k_i's are non-zero. Such a representation is called a NAF representation.

2. Show that the standard binary representation of an integer can be converted into a NAF representation.

3. Find the standard binary representation and NAF representation of 749.

4. Show that a signed representation of a positive integer in NAF form is unique.