Problem of the Week-1: A Recursive Function

A function f is defined for all positive integers and satisfies

$$f(1) = 2011, \text{ and } f(1) + f(2) + \cdots + f(n) = n^2 f(n) \text{ for all } n > 1$$

Find the exact value of $f(2011)$.

As always, explain and justify your answer.