Problem 3: All 1's

Let n be a positive integer that is not divisible by 2 or 5. Show that there is a multiple of n whose digits are all 1's.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.