Problem 2: Maximizing an Integral

Let C be the set of all continuous real-valued functions $f : [0,1] \to \mathbb{R}$ satisfying

$$|f(x) - f(y)| \leq |x - y|, \qquad 0 \leq x, y \leq 1, \qquad f(0) = 0.$$

Let $\phi : C \to \mathbb{R}$ be defined by $\phi(f) = \displaystyle\int_0^1 (f(x)^2 - f(x))dx$. Show that ϕ attains its maximum at some element of C.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 9/5/2020. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by 4 pm on Sep 18, 2020. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.