Problem 1: A Polynomial Identity

Find all polynomials satisfying the identity

$$p(x + 1) = p(x) + 2x + 1$$

for all $x \in \mathbb{R}$. Make sure to justify how you know you have found all such polynomials.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.