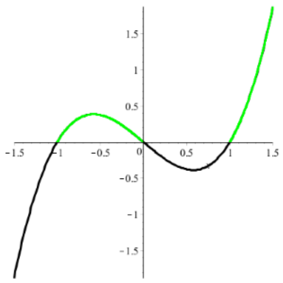
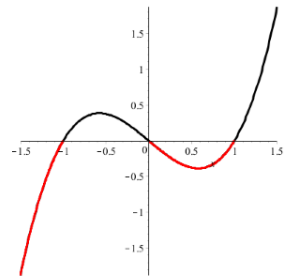
Geometry of Graphs
Vocabulary

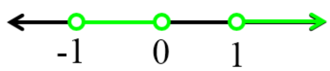
<u>Important terms for describing the behavior a function *f*:</u>

positive/negative: *f* is positive at the point *x* if $f(x) > 0$ and *f* is negative at the point *x* if $f(x) < 0$. (Similarly: non-negative means $f(x) \geq 0$ and non-positive means $f(x) \leq 0$.)

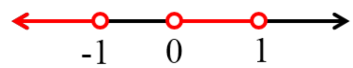
For instance, if we are considering the function $f(x) = x^3 - x$, we have



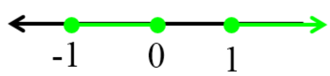




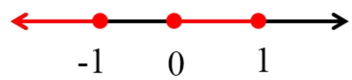
f is **positive** on (-1,0) and on (1,∞).



f is **negative** on (- ∞,-1) and on (0,1).

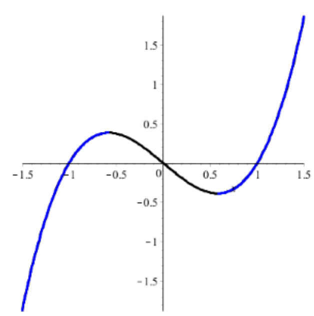


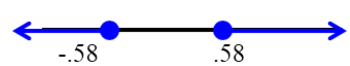
f is **non-negative** on [-1,0] and on [1,∞).



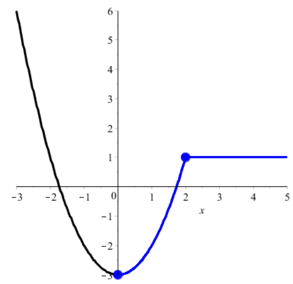
f is **non-positive** on (-∞,-1] and on [0,1].

Increasing/decreasing: *f* is **strictly increasing** on the interval (*a*,*b*) provided that if you move from left to right on the graph, you go uphill. Formally: if $x_1 < x_2$, then $f(x_1) < f(x_2)$. *f* is **increasing** on the interval (*a*,*b*) provided that if you move from left to right on the graph, you don't go downhill: Formally: if $x_1 < x_2$, then $f(x_1) \leq f(x_2)$. Strictly increasing functions are increasing; the reverse is not true. (The diagram below shows a function that is increasing on [0,∞), but not strictly increasing.)



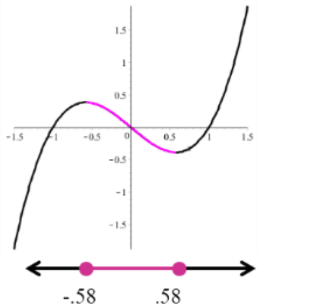


f is **strictly increasing** on (approximately) (- ∞,-.58) and on (.58, ∞)



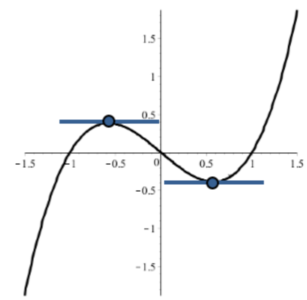
6
5
4
3
2
1
−3
−2
−1
0
1
2
3
4
5
x
−1
−2
−3

Similarly, *f* is **strictly decreasing** on the interval (*a*,*b*) provided that if you move from left to right on the graph, you go downhill. Formally: if $x_1 < x_2$, then $f(x_1) > f(x_2)$. *f* is **decreasing** on the interval (*a*,*b*) provided that if you move from left to right on the graph, you don't go uphill.



f is **strictly decreasing** on (approximately) (- .58,.58)

Stationary point: a point *x* is a stationary point for the function *f*, provided that $f'(x) = 0$.



f has stationary points at (approximately) *x* = -.58 and *x* = .58.

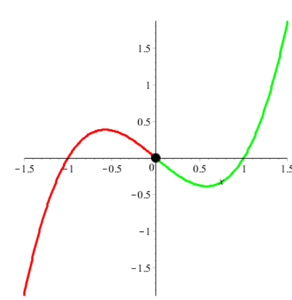
Graphical Characteristics of ("nice") functions

Other important terms: concave up/ concave down (on an interval) (Rather than defining this in a formal mathematical way, we just think of this graphically.)

Four "puzzle pieces":

Increasing Concave Up	Increasing Concave down	Decreasing Concave Up	Decreasing Concave down

inflection point: *x* is an inflection point for the function *f*, provided that ***f* changes concavity** at *x*. That is, as the graph of *f* moves through the point $(x, f(x))$, the graph goes from concave up to concave down or vice versa. ($f(x) = x^3 - x$ has a point of inflection at *x* = 0, because its graph goes from concave down to concave up there, as shown in the graph to the right.)



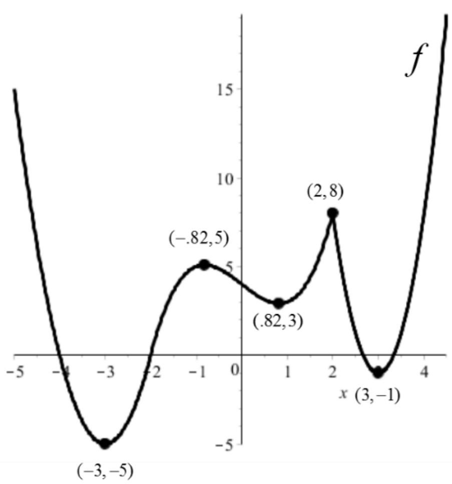
Local and Global Extrema

Maxima: A function *f*

- has a **global** or **absolute maximum** at x = *a* provided that there is no point on the graph of *f* that is higher than $(a, f(a))$.
- has a **local maximum** at x = *a* provided that $(a, f(a))$ is the highest point in a small region of the graph surrounding it.

Minima: A function *f*

- has a **global** or **absolute minimum** at x = *a* provided that there is no point on the graph of *f* that is lower than $(a, f(a))$.
- has a **local minimum** at x = *a* provided that $(a, f(a))$ is the lowest point in a small region of the graph surrounding it.



For some examples, consider the function f whose graph is shown at the left,

- f has local maxima at (approximately) $x = -.82$ and at $x = 2$.
- f does not have a global maximum.
- f has local minima at $x = -3$, (approximately) $x = .82$ and at $x = 3$.
- $x = -3$ is also a global minimum for *f*.

The generic word for either maximum or minimum is **extremum**.

General concept		
Maximum Plural: maxima	**Local** maximum (or maxima)	**Global** or **absolute** maximum or (maxima)
Minimum Plural: minima	**Local** minimum (or minima)	**Global** or **absolute** minimum or (minima)
Extremum Plural: extrema	**Local** extremum (or extrema)	**Global** or **absolute** extremum or (extrema)