IBL Prep Workshop
June, 2008

Problems and Scenarios

Scenario 1: You are teaching a real analysis class and have just defined continuity.

Definition: Let X and Y be metric spaces. Let $f : X \to Y$ be a function. We say that f is continuous at the point x in X if for all $\epsilon > 0$, there exists $\delta > 0$ such that if $d_X(x, y) < \delta$, then $d_Y(f(x), f(y)) < \epsilon$.

Your students have been told that K is a fixed real number, that x is a fixed element of the metric space X, and that $f : X \to \mathbb{R}$ is a continuous function. They have been asked to prove that if $f(x) > K$, then there exists an open ball about x such that f maps every element of the open ball to some number greater than K.

One of your students comes into your office saying that he "has tried everything" but cannot make headway on this problem. When you ask him what exactly he has tried, he simply reiterates that he has tried "everything." What is going on? What do you do?

Scenario 2: You have just defined subspace (of a vector space) in your linear algebra class.

Definition: Let V be a vector space. A subset S of V is called a subspace of V if S is closed under vector addition and scalar multiplication.

The obvious thing to do is to try to see what the definition means in $\mathbb{R}^2$ and $\mathbb{R}^3$. You could show your students, but you would rather let them play with the definition and discover the ideas themselves. Design a class activity that will help the students classify the linear subspaces of 2 and 3 dimensional Euclidean space. (You might think about "separating out the distinct issues.")

Scenario 3: Your students are studying some basic set theory. They have already proved DeMorgan's laws for two sets. (And they really didn't have too much trouble with them.) You now want to generalize the proof to an arbitrary collection of sets. That is

$$\bigcup_{\alpha\in\Lambda} A_\alpha^{\mathcal{C}} = \left(\bigcap_{\alpha\in\Lambda} A_\alpha\right)^{\mathcal{C}} \text{ and } \bigcap_{\alpha\in\Lambda} A_\alpha^{\mathcal{C}} = \left(\bigcup_{\alpha\in\Lambda} A_\alpha\right)^{\mathcal{C}}.$$

The argument is the same, but your students are really having trouble. What's the root of the problem? What should you do?

Scenario 4: A very good student walks into your office. She has been asked to prove that the function

$$f(x) = \frac{x}{1+x}$$

is one-to-one on the interval $(-1, \infty)$. She says that she has tried but can't do the problem. This baffles you because you know that just the other day she gave a lovely presentation in class showing that the composition of two one-to-one functions is one-to-one. What is going on? What should you do?

Scenario 5: Your students are studying partially ordered sets. You have just introduced the following definitions:

Definition: Let $(A, \leq)$ be a partially ordered set. Let x be an element of A. We say that x is a **maximal element** of A if there is no $y \in A$ such that $y > x$. We say that x is the **greatest element** of A if $x \geq y$ for all $y \in A$.

Anecdotal evidence suggest that about 71.8% of students think these definitions say the same thing. *Why do you think this is?* Design a class activity that will help your students differentiate between the two concepts. While you are at it, build a way for them to see why we use "a" when defining maximal elements and "the" when defining greatest elements.