Open Questions

Important questions arose, and remain unanswered, after several years' work in preparation of this *Guide.* We collect several such questions here. Exploring some of them will require large studies and financial support; we expect CUPM and other committees of MAA and other CBMS organizations to investigate some of them. Other questions may prompt individuals in our community to explore and experiment.

1. Getting started in college mathematics

a. *Broader bridges to further mathematics.* A persistent question for CUPM and its subcommittee, CRAFTY, is what mathematics courses to recommend or design for students entering the university. There is interest in offering courses that present a broad view of the subject and make students aware of the wide variety of disciplines that benefit from mathematical thinking and techniques. These courses should serve as bridges to courses that serve both mathematics majors and others. We hope to collect examples of innovative introductory courses distinct from and independent of college algebra and calculus. Several such courses exist, but they are seldom seen as leading to further mathematics courses, let alone to a major. Many institutions now require First-Year Seminars in various areas; CUPM expects to compile a list of such seminars with mathematical themes. Could these seminars provide a bridge to the major?

b. *Discrete mathematics courses.* There was an effort in the 1980s to promote first-year discrete mathematics courses as alternatives to elementary calculus. Recommendations from this effort remain timely. Should we revive this effort? Can discrete mathematics serve as an entry point to a mathematics major? Together with calculus, discrete mathematics would prepare students for further work in the social and behavioral sciences, computer science, information technology, and business.

c. *Serving underprepared students.* How can we best serve the many students who arrive underprepared for college mathematics? Experiments with "stretch courses" are ongoing. Credit-bearing college courses in calculus were expanded to include developmental topics on an as-needed basis in the 1980's. Newer examples of courses outside the standard pre-calculus track designed to incorporate such developmental mathematics in credit-bearing courses are being designed and tested. Are such efforts successful? Can we scale them up?

d. *Data analysis.* The American Statistical Association recommends that *all* students take a course in data analysis. CUPM recommends such a course for all mathematical sciences majors. If this is the first mathematics-related course that new college students take, what are the implications? Will such courses satisfy university mathematics requirements? What developmental mathematics will students need to succeed? Will calculus (and pre-calculus) courses remain preferred for beginning STEM majors? How well does the AP Statistics examination reflect the content of a college data analysis course? Who will staff such courses? What can we expect from beginning college students who have been prepared in programs based on the Common Core State Standards for Mathematics? The CCSSM recommends added emphasis on probability and statistics; will that change the recommended college-level data analysis course? What professional development will faculty need?

e. *Emphasis on modeling.* There have been repeated calls for a modeling course at the college algebra and pre-calculus levels, and textbooks have appeared. What is meant by modeling? We are struggling to define the term as it applies to various levels of mathematics courses. How common and popular are modeling courses at the entry level? Do they successfully prepare students for a next course in mathematics? What should the next course(s) be for biology, chemistry, and other STEM majors?

f. *Persistence in the pipeline.* How can we best encourage students who succeeded in high school calculus to take college mathematics and consider a mathematics major?

2. **Calculus**

a. *National studies.* Following on the MAA study of Calculus I, *Characteristics of Successful Programs in College Calculus* , we need to know more about the success and persistence of potential STEM majors across the entire precalculus through calculus sequence as well as adequacy of their preparation for downstream courses, both in mathematics and in other disciplines. The new NSF-funded MAA study, *Progress through Calculus*, will address some of these issues.

b. *Restructuring college calculus.* Several efforts are underway to redesign beginning college calculus courses to interest and challenge students who have met calculus ideas in high school. Can we describe syllabi and goals of these alternative courses and measure their success? How transportable are these courses to other institutions?

c. *Serving biology majors.* Should the large and growing population of biology majors take the same introductory mathematics courses as other STEM majors? Perhaps these students' early experience should include elements of modeling, data analysis, and linear algebra—as well as calculus.

d. *Blended calculus.* What examples exist of successful team-taught interdisciplinary courses, blending calculus with other subjects such as physics or biology?

3. **Technology: an evolving challenge**

   a. *"Smart" technologies*. How does—and will—the ubiquity of computer algebra systems, tablets, smart phones, and sophisticated calculators affect the teaching and learning of mathematical content? What concepts and techniques must students master? What does mastery mean, and how should we assess it?

   b. *Technology in the classroom.* How does, and how should, technology change the ways mathematical material is presented in class?

   c. *Online resources.* How can we use online resources (You Tube lectures, MOOCs, etc.) to make students' experiences in mathematics more interesting and successful? This question may be especially significant for large, first-year courses.

   d. *Technology and learning.* How can and should technology change the ways students learn and how they mature mathematically?

4. **"High-impact" learning experiences**

   a. *Capstone courses.* What makes an effective capstone course? CUPM continues to call for examples of senior-level capstone courses. Examples will ideally include course objectives, syllabi, requirements, and evaluation of course effectiveness.

   b. *Research and research-like experiences*. Many different strategies exist to offer undergraduates research or research-like experiences. Such offerings are often expensive; we need new models and alternatives that allow departments to offer such experiences at larger scale. What are alternative ways for students to participate? Can team projects with one advisor and several students provide the desired experience? Might industrial or business sponsors rather than full-time faculty direct team projects that explore applied problems ?

c. *Internships.* Internships are popular among students. How can the mathematical community encourage partners in business, industry, and government to sponsor more and mathematically richer internships? How can information be shared with advisors and students?

5. **Teaching for cognitive growth**

a. *Achieving cognitive goals.* What are our best pedagogical strategies for helping students achieve the cognitive goals set out in the Overview? What evidence supports these "best" strategies? How can we help departments design courses that intentionally build students' mathematical sophistication and maturity throughout their undergraduate programs?

b. *Encouraging innovation.* Faculty—including junior faculty— should be encouraged to experiment with new pedagogy and content, without feeling unduly at risk in the promotion and tenure processes. Faculty should be accountable for their experiments, but they should also be invited to document their work and provide evidence for what works well or poorly. Departments should foster a culture that rewards innovation—and accepts that not all innovations succeed.

c. *Setting standards.* Can the mathematics community work toward agreement on ways to measure pedagogical effectiveness ?

6. **Professional rewards and course development**

a. *Encouraging course development*. How can the mathematical community support course development? Individuals in many institutions are developing materials that change their own courses. Many good ideas emerge. How can we encourage departments to encourage and reward innovative development of content?

b. Defining alternative pathways into the major. Can we define new pathways into the mathematics major that can be widely applied? What are reasonable routes distinct from the traditional entry from calculus?

c. Scaling up good ideas. How do we communicate with each other to promote broader adoption of new ideas?

d. Facilitating restructuring large-enrollment course sequences. Can the community effectively help departments restructure their large-enrollment course sequences when we have to consider transfer students, other disciplines, the nature of the teaching corps? How do we generate cooperation and enthusiasm for such changes?

e. *Rewarding efforts in reform.* Do departments have internal mechanisms for professional rewards for those who develop, communicate, and inspire curriculum reform?

7. **Building interdisciplinary cooperation**

Many of our colleagues in other departments see us as isolated loners. Mathematicians should meet regularly with colleagues in other disciplines to advance common interests. What strategies have succeeded? How can institutions avoid the silo effect? How can young faculty be encouraged to initiate and sustain interdisciplinary relationships? How can the mathematical community support these efforts?

8. **Professional development**

a. *Supporting adjunct faculty*. Reliance on non-tenured, non-tenure track adjunct faculty to teach at the introductory level is common at many institutions, and likely to continue. It can be difficult to change curricula, to design innovative courses, and to monitor success when, as at some institutions, adjunct faculty teach more than 50% of students in their first two years. How can departments support professional development for adjuncts?

b. *Supporting permanent faculty.* Tenured and tenure-track faculty may be set in their ways, eager to pursue their own research (for which they are rewarded), and reluctant to try time-consuming new approaches. What are effective ways to change attitudes and reward systems to encourage professional development, new ideas, and practices?

9. **Articulation**

How can the professional national organizations and the statewide mathematics associations work with high schools to better communicate what students will know when they reach college? What does it really mean to be "college and career ready"? Does the same standard apply to everyone? How will the Common Core State Standards for Mathematics (CCSSM) change our students' mathematical background? How should placement testing reflect these changes? It seems inane to say that college mathematics departments should make themselves familiar with the CCSSM. But many myths about the Standards persist. Are changes needed in the placement system in colleges, or in the CCSSM itself, to help our students succeed in college-level mathematics? Does the college-level mathematics that we offer have to change?

10. **Increasing diversity, broadening interest**

   a. How can departments increase the diversity of students in mathematics courses?

   b. How can we best scale up already successful programs for supporting minorities and women in mathematics?

   c. How can we better advocate for the importance of mathematics in all fields and applications? How can we better convince parents, high school students, and legislators that mathematics matters to all of us, and deserves support?

Where do we go from here?

This *Guide* is inevitably incomplete, and its recommendations likely to change over time. The curriculum is always evolving—inevitably, as mathematics grows and its applications develop. Still, we hope that the mathematical community, together with its broad academic partners, will engage in discussion and build consensus on curriculum as we move forward. The Internet can help us negotiate our rapidly changing environment by supporting the exchange of new ideas and evidenced-based models.

How can the mathematical community identify good ideas that have been successfully implemented? How can we measure success? What mechanism is available for scaling up the curricular and pedagogical innovations that seem to be effective?

CUPM welcomes the challenge of starting virtual communities dedicated to improving, supplementing, and evaluating its recommendations. It is an exciting, if awesome, task. What more can the mathematical community offer?