Problem of the Week — November 9, 2009

A certain bug named Nat is taking a *lattice-walk* under the curve $y = \sqrt{x}$. Nat's first few steps (or lattice visits) on his walk are

$$(0,0), (1,0), (1,1), (2,0), (2,1), (3,0), ...$$

As is evident, Nat's walk proceeds by the following simple rules. From a lattice point $(a, 0)$, he ascends vertically, visiting the lattice points $(a, 0), (a, 1), (a, 2), ...$, etc, until he reaches the curve $y = \sqrt{x}$. Upon encountering the curve, Nat drops down to $(a + 1, 0)$ without visiting any lattice points on the way down, and continues his walk by climbing the lattice again, visiting $(a+1, 0), (a+1, 1), (a+1, 2), ...$, etc, until he once more encounters the curve, whereupon he drops down to $(a + 2, 0)$ and begins another ascent. Nat continues his lattice walk under $y = \sqrt{x}$ in this way indefinitely. Today Nat is celebrating the occasion of visiting his 200990022009^{th} lattice point since starting his walk at $(0, 0)$ (call $(0, 0)$ his first lattice visit) . What are Nat's coordinates on this special occasion? Naturally, you must prove your result.

Solutions accepted until 4 pm 11/20/09

You may submit complete solutions to Brian Jones or Marie Snipes either via email or hard copy; however, if you submit a hard copy, it must have a time-stamp (i.e. either electronic proof of time printed or a faculty signature verifying the time submitted.)