

3. (6 points) Determine whether the set $\left\{ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ is linearly independent in M_{22} .

4. (12 points) Let $T : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$ be given by $T(x, y, z) = (2x - y, 2x + y, x + z)$.

(a) Is T a linear transformation? If so, what is $[T]$, the standard matrix representation of T ?

(b) Does T^{-1} exist? If so, what is $[T^{-1}]$?

5. (20 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$.

- (a) Calculate the characteristic polynomial of A . You may use Maple or Mathematica or a calculator to compute a determinant, but please do not simply ask for the characteristic polynomial of A .

- (b) Calculate the eigenvalues of A by factoring the characteristic polynomial of A .

(c) Calculate bases for the eigenspaces of A by row reduction of the appropriate matrices.

(d) What are the algebraic and geometric multiplicity of each eigenvalue of A ?

6. (6 points) By what matrix would you multiply a vector \vec{v} to change it from the standard basis to the basis $B = \{(1, 0, 1), (0, -1, 2), (2, 3, -5)\}$?

7. (8 points) Let W be a subspace of inner product space V . Show that W^\perp is a subspace of V .