

Math 224 Exam #2
2 November 2006

Name: _____

INSTRUCTIONS: Complete the following problems without the aid of any person or any book. You may use a calculator or a computer algebra system such as Maple or Mathematica.

1. (8 points) Let W be a non-empty subset of vector space V . Prove that if W satisfies vector space axioms 1 and 6, then W is a vector space.

2. (6 points) Do the set of pairs (p, q) such that p and q are rational form a vector space? (Assume the usual rules for addition and scalar multiplication on pairs.) Justify your answer.

3. (9 points) Is $\{x^2 - 4x + 4, 2x^2 - x + 1, x^2 - 2x + 2\}$ linearly dependent or linearly independent in P_2 ? If it is linearly dependent, write one of the polynomials as a linear combination of the other two. You are free to use Maple/Mathematica/a calculator to calculate with a matrix, but let me know what you used the software to do.

4. (12 points) Complete the following definitions:

(a) W is a *subspace* of vector space V if

(b) Vector \vec{v} is in the *span* of set S of vectors if

(c) Set S of vectors are *linearly independent* if

(d) Set B of vectors from vector space V forms a *basis* for V if

(e) Vector space V is *infinite-dimensional* if

(f) Vector space V has *dimension* n if

5. (6 points) Let V have dimension n .

(a) If set S of vectors from V has more than n vectors, then...

(b) If set S of vectors from V has fewer than n vectors, then...

6. Let $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$. The reduced row-echelon form of A is $\begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

- (a) (2 points) What is the rank of A ?
- (b) (2 points) What is the nullity of A ?
- (c) (3 points) Find a basis for the row space of A .

- (d) (3 points) Find a basis for the column space of A .

(e) (3 points) Find a basis for the nullspace of A .

(f) (3 points) Given $A \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 19 \\ 21 \\ 47 \end{bmatrix}$, find a general form of all solutions to $A\vec{x} = \begin{bmatrix} 8 \\ 19 \\ 21 \\ 47 \end{bmatrix}$.

7. (8 points) Assume that A is a 4×7 matrix.

- (a) The rank of A is between _____ and _____.
- (b) The nullity of A is between _____ and _____.
- (c) The rank of A^T is between _____ and _____.
- (d) The nullity of A^T is between _____ and _____.

8. (6 points) Let $\vec{u} = (0, 1, 2, 3)$ and let $\vec{v} = (1, 0, 4, -1)$.

(a) Compute $\vec{u} \cdot \vec{v}$.

(b) Compute $\|\vec{u}\|$.

(c) Compute $d(\vec{u}, \vec{v})$.

9. (3 points) State the Cauchy-Schwartz inequality.

10. (2 points) State either form of the triangle inequality.