Final Exam

Part I: No Maple

Math 224: Linear Algebra Name: ____________________

21 points

• You must show all work to receive full credit.

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1. (7 pts) Determine the general solution of

$$\begin{aligned} x + 3y + 2z &= 6 \\ 2x + y &= 3 \\ 2y + z &= 0 \end{aligned}$$

using either the Guassian Method or the Guass-Jordan Method.

2. (7 pts) Calculate the inverse of

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$$

provided it exists.

3. (7 pts) Use Cramer's Rule to calculate z for the system

$$\begin{aligned} x + 4z &= 2 \\ 2x + y - z &= 3 \\ y + z &= 1. \end{aligned}$$

Final Exam

Part II: Maple may be used.

Math 224: Linear Algebra Name: ____________________

79 points

- You must show all work to receive full credit.

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1. (10 pts) Determine if the set $H = \{(x, y, z) : x - 4y - z = 0\}$ is a subspace of $\Re^3$.

2. (10 pts) Let $w_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 3 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 1 \end{pmatrix}$, and $w_4 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 2 \end{pmatrix}$. Find a basis for the Span(w_1, w_2, w_3, w_4) and the dimension of the space.

3. (15 pts) Let

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

a. Find a basis for the column space of A.

b. Find a basis for the row space of A.

c.Find a basis for the null space of A.

3. (5 pts) Let $A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & -1 & 4 \\ -4 & 5 & -10 & -6 \\ 3 & -2 & 10 & -1 \end{pmatrix}$. Determine the rank($A$) and the nullity($A$).

4. (5 pts) Determine if the matrices $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are linearly independent in M_{22}.

5. (9 pts) Let $T : \Re^3 \to \Re^3$ be given by $T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} 2x - y \\ 2x + y \\ x + z \end{pmatrix}$.

a. Is T a linear transformation? If so, give the standard matrix representation of T.

b. Does T^{-1} exist. If so, give the standard matrix representation of T^{-1}.

6. (15 pts) Let $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{pmatrix}$.

a. Give the characteristic polynomial of A.

b. Calculate the eigenvalues of A.

c. Determine the corresponding eigenspace for each eigenvalue.

d. Give the algebraic multiplicity and the geometric multiplicity of each eigenvalue.

7. (10 pts) Let $\mathbf{p} = p(x)$ and $\mathbf{q} = q(x)$ be polynomials in P_2. Define

$$\langle \mathbf{p}, \mathbf{q} \rangle = p(0)q(0) + p(1)q(1) + p(2)q(2).$$

a. Show that $\langle \mathbf{p}, \mathbf{q} \rangle$ defines an inner product on P_2.

b. Give a general formula for $\|\mathbf{p}\|$.