Group Work 2, Section 7.2
$\|A\|_2$ Guessing

Choose a symmetric 2×2 matrix A. To compute $\|A\|_2$ we will need to know the maximum stretching factor of A when applied to the unit vectors.

1. Explain why every unit vector in R^2 can be written as $\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ for some $t \in [0, 2\pi)$.

2. For $t \in [0, 2\pi)$, let $\mathbf{v} = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ and compute $A\mathbf{v}$. Your answer should be a 2×1 matrix where each entry is a function of t. Let $v_1(t)$ denote the $(1, 1)$ entry of $A\mathbf{v}$ and $v_2(t)$ denote the $(2, 1)$ entry of $A\mathbf{v}$.

3. Let $f(t) = \sqrt{v_1(t)^2 + v_2(t)^2}$. Find the approximate maximum value of this function on $[0, 2\pi)$. This a good place to use a graphing device!

4. The maximum value you found in Question 3 is the value of $\|A\|_2$. Now find the eigenvalues of A.

5. What is the relationship between the eigenvalues of A and $\|A\|_2$?