Math 224: Linear Algebra
Midterm Exam 2
November 5, 2009

No Maple or calculators are allowed on part 1 of this exam. Once you have finished this part you may turn it in for the rest of the exam, which you may complete with the aid of Maple or calculators. Please show all your work for each problem! You may use any method to solve these problems but you need to explain your reasoning clearly.

1. (12 pts)

   (a) Find the eigenvalues and associated eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

   (b) Sketch the eigenspaces of A.

2. (4 pts) Compute the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 6 & 3 & 7 \end{bmatrix}$.

3. (10 pts) Find the inverses of the following two matrices, if they exist. If an inverse doesn't exist, briefly explain how you know.

(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 6 & 3 & 7 \end{bmatrix}.$$

(b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}.$$

4. (16 pts) Determine whether each of the following sets S of vectors in $\mathbb{R}^4$ is a subspace of $\mathbb{R}^4$. For each set, either show (using the definition of a subspace) that the set forms a subspace or give a counterexample to show that it is not.

(a) $S = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} : v_1 > 0 \right\}$

(b) $S = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} : \pi v_2 - v_3 = v_4 \right\}$

(c) $S = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} : v_1 v_2 = 0 \right\}$

(d) $S = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} : v_4 \text{ is rational} \right\}$

5. (10 pts) The plane in $\mathbb{R}^3$ containing the points $(0,0,0)$, $(4,5,0)$, $(6,8,2)$, and $(10,15,10)$ is a subspace of $\mathbb{R}^3$. Find a basis for this subspace.

6. (10 pts) (a) If A is a 7×4 matrix (i.e., A has 7 rows and 4 columns), explain why the rows of A must be linearly dependent.

(b) What are the possible values for nullity (A)?

7. (6 pts) Let A be the standard matrix representation of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$. Describe geometrically what T does to a vector in $\mathbb{R}^2$.

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

8. (12 pts) Answer the following questions for the matrix M below.

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

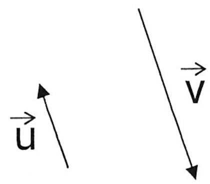
(a) What is the dimension of row(M)?

(b) Find a basis for row(M).

(c) Circle all vectors below that are in null(M).

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 42 \\ -12 \\ -18 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

9. (6 pts) Suppose that the vector $\vec{u}$ is an eigenvector of the matrix A and $\vec{v} = A^3\vec{u}$. If $\vec{u}$ and $\vec{v}$ are as shown in the picture below, provide a reasonable *estimate* the eigenvalue of A corresponding to $\vec{u}$. Explain your reasoning.



10. (14 pts) What is the standard matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ for which

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \text{ and } \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}?$$

11. (2 pts EXTRA CREDIT!) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^3$.

Write the coordinate vector of $\vec{v} = \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$ with respect to $\mathcal{B}$. In other words, find $[\vec{v}]_{\mathcal{B}}$.