Group Work 5, Section 1.2
Find the Error (Part 1)

It is a beautiful spring morning. You are about to go to your 4 P.M. class, but have stopped at a convenience store to buy carrot sticks and bottled water for a healthy snack. As you wait in line to pay for your purchases, whistling to yourself, you notice a wild-eyed gentleman standing in line in front of you, buying a moon pie.

"Well aren't you a merry grig?" he asks. You nod noncommittally, since you have no idea what a "grig" is. He takes your nod to mean that you would like further conversation, and asks, "Where are you off to now?"

"Why, I'm off to my linear algebra class, to learn some useful information about vectors."

"Vectors, vectors," he says, half to himself. "I remember learning about vectors... I remember learning... LIES!"

"What do you mean, 'lies'?" you ask. "Everything we've learned about vectors is as true as it is useful."

"Oh yes? You think you know it all, do you?" By this point, he has paid for his purchases. As you pay for yours, you notice him writing on his receipt:

Let $\mathbf{u}$ be a vector such that $\|\mathbf{u}\| = \sqrt{2}$. Choose a vector $\mathbf{v} \neq \mathbf{u}$ such that $\mathbf{u} \cdot \mathbf{v} = 2$. Now we have

$$2\,(\mathbf{u} \cdot \mathbf{u}) \;=\; \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v}$$
$$2\,(\mathbf{u} \cdot \mathbf{u}) - 2\,(\mathbf{u} \cdot \mathbf{v}) \;=\; \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} - \mathbf{2}\,(\mathbf{u} \cdot \mathbf{v})$$
$$2\,(\mathbf{u} \cdot \mathbf{u}) - 2\,(\mathbf{u} \cdot \mathbf{v}) \;=\; \mathbf{u} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v}$$
$$2\mathbf{u} \cdot (\mathbf{u} - \mathbf{v}) \;=\; \mathbf{u} \cdot (\mathbf{u} - \mathbf{v})$$
$$2\mathbf{u} \;=\; \mathbf{u}$$
$$2 \;=\; 1$$

"I've seen someone try this before," you say dismissively, "in college algebra. But you are not allowed to divide by zero."
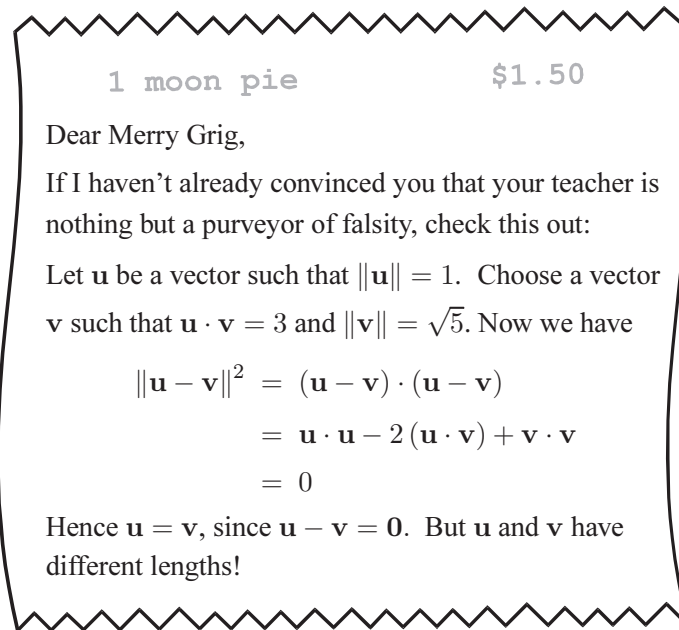
"Ah, but I am not dividing by zero! Since $\mathbf{u} \neq \mathbf{v}$, we know that $\mathbf{u} - \mathbf{v}$ cannot be zero! Now you go enjoy your class, while I go and enjoy my moony pie!" And the stranger leaves, singing a strange song to himself, and opening the wrapper to his moon pie.

Could Linear Algebra be flawed already? Two can't equal one, can it? Find the error in the gentleman's reasoning.

Group Work 6, Section 1.2
Find the Error (Part 2)

After determining the stranger's mistake, you go to your Linear Algebra class. Your teacher tells you to pay particular attention to page 18, so you take a scrap of paper to mark the page. You notice that you are using the gentleman's receipt, and that he has written something on the front of the receipt as well!

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1 moon pie                    $1.50
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Dear Merry Grig,

If I haven't already convinced you that your teacher is nothing but a purveyor of falsity, check this out:

Let $\mathbf{u}$ be a vector such that $\|\mathbf{u}\| = 1$. Choose a vector $\mathbf{v}$ such that $\mathbf{u} \cdot \mathbf{v} = 3$ and $\|\mathbf{v}\| = \sqrt{5}$. Now we have

$$
\begin{aligned}
\|\mathbf{u} - \mathbf{v}\|^2 &= (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\
&= \mathbf{u} \cdot \mathbf{u} - 2(\mathbf{u} \cdot \mathbf{v}) + \mathbf{v} \cdot \mathbf{v} \\
&= 0
\end{aligned}
$$

Hence $\mathbf{u} = \mathbf{v}$, since $\mathbf{u} - \mathbf{v} = \mathbf{0}$. But $\mathbf{u}$ and $\mathbf{v}$ have different lengths!

Well, gosh darn him anyway! How can two things be the same, and yet different?

Find the error.