Math 213 - A Vector Primer

The material covered in Sections 13.1 and 13.2 will be a review for some of you and completely new for others. Please keep this in mind if you feel completely bored (or terrified!) in working through this primer. If you need additional help in understanding the material, do not hesitate to come to my office with your questions.

1. For parts a) through e) below, assume that $\vec{u} = \langle 1, 3 \rangle$, $\vec{v} = \langle 2, -1 \rangle$, and $\vec{w} = \langle -2, 2 \rangle$.

a) Compute $\vec{u} + \vec{v} - \vec{w}$.

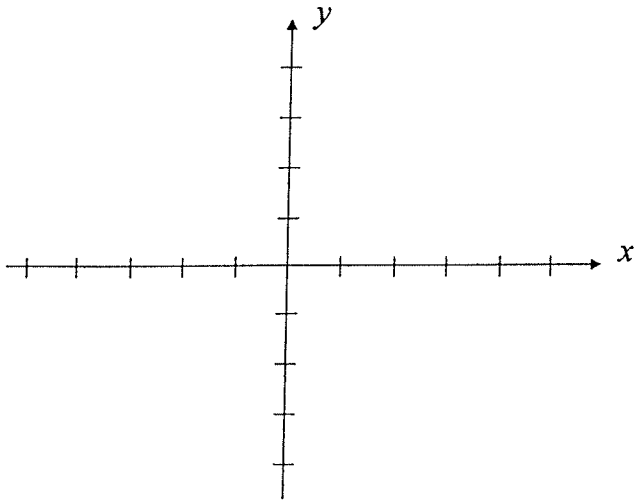
b) Compute $3\vec{u} - 2\vec{v}$.

c) Make $\vec{w}$ into a unit vector. That is, find the vector of length one pointing in the same direction as $\vec{w}$. (We denote this vector by $\hat{w}$.)

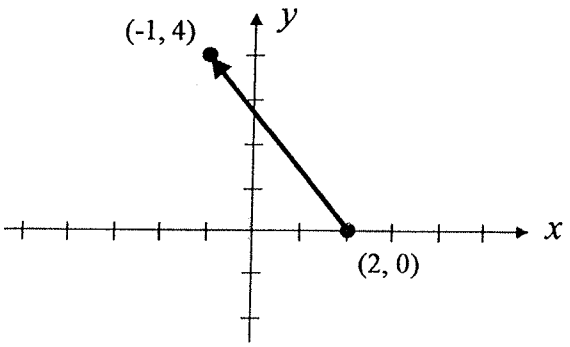
Remark: Unit vectors are very important. We use them when we need to describe direction only (not magnitude).

d) Find the vector of length 3 pointing in the same direction as $\vec{v}$.

e) Sketch the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$, $\vec{u} + \vec{v}$, and $\vec{u} - \vec{w}$ on the axes below.

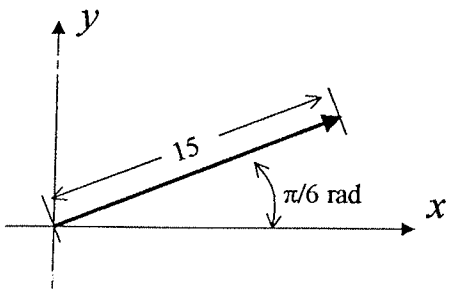


2. a) The vector pictured below can be described as the **displacement vector** from (2, 0) to (-1, 4). Resolve this vector into its components. That is, express the vector in the form $\vec{v} = v_1\hat{i} + v_2\hat{j} = \langle v_1, v_2 \rangle$.

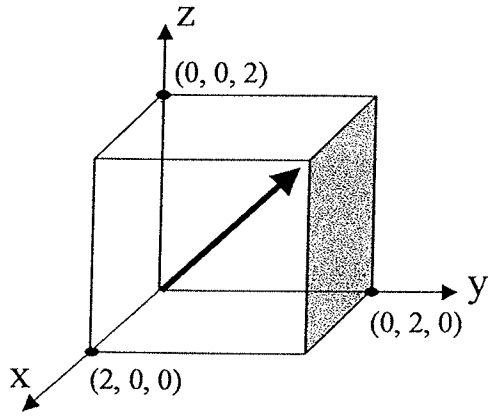


b) What is the displacement vector with tail at (1, 2, 3) and head at (4, -1, 0)?

3. Resolve the vector pictured below into its components.



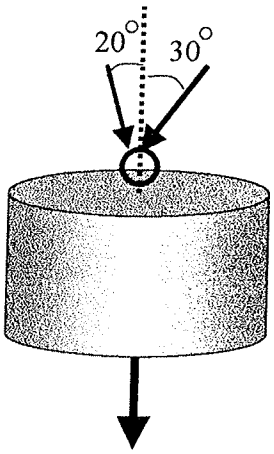
4. Find the unit vector $\hat{v}$ in the directon of the diagonal of the cube shown below.



5. a) An airplane is traveling at a fixed altitude with a negligible wind factor. The plane is headed N 30° W (which means 30° west of north) at a speed of 500 miles per hour. Find the vertical (North/South) and horizontal (East/West) components of the plane's velocity vector.

b) As the plane (described in part a) reaches a certain point, it encounters wind with a velocity of 70 miles per hour in the direction E 45° N. What is the resultant speed and direction of the plane?

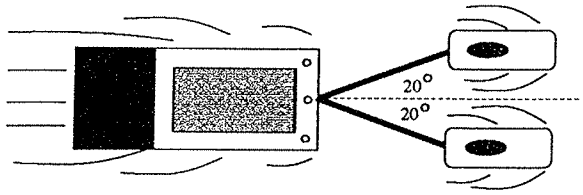
6. To carry a 100 pound cylindrical weight, two workers lift on the ends of short ropes tied to an eyelit on the top center of the cylinder. One rope makes a 20° angle away from the vertical and the other a 30° angle (see the figure below).



a) Find the tension in each rope if the resultant force is vertical. (Tension is simply the magnitude of the force vector.)

b) Find the vertical component of each worker's force.

7. A loaded barge is being towed by two tugboats, and the magnitude of the resultant is 6000 pounds directed along the axis of the barge. Find the tension in the tow lines if they each make a 20 degree angle with the axis of the barge.



8. Use vectors to find the points of trisection of the line segment with endpoints (1,2) and (7, 5).

9. Prove that the vectors $\vec{u} = \cos\theta\hat{i} - \sin\theta\hat{j}$ and $\vec{v} = \sin\theta\hat{i} + \cos\theta\hat{j}$ are unit vectors for any angle θ.