Writing Assignment #1 – Exploration of Limits
Due Date: Wednesday, September 14

Attached is your first writing project of the semester. On your own paper, complete each of the five exercises ***paying close attention to your presentation***. We are actually less interested in your final answer than we are in how you present your reasoning. To this end, you should include graphics to illustrate your ideas and reasoning (plot the relevant surface and/or contour diagrams with *Maple*, and print it off). When answering a question, write in complete sentences. Show all steps of your computations and use proper notation. When computing limits along specific paths, you will need to compute them by hand. It is not enough to provide *Maple's* output[1]. Please note that you are required to type up your work. You can either use *Microsoft Word* or Maple. *Word*'s "Equation editor" will allow you to include fancy equations and notation, and the program allows you to import *Maple* graphics. Ask your professor for help if you do not know how to do this. Alternatively, you can use Maple as a text editor, inserting mathematical equations and graphs as needed. Finally, you should be neat and concise. You will be penalized if you include misspellings and improper punctuation in your write-ups. You may work in pairs if you like, but both people need to be involved in all aspects of the work and there can be no more than two to a group. The assignment is due on Wednesday, September 14.

EXPLORATION OF LIMITS

Exercise 1. Plot the surface $z = \dfrac{2xy}{x^2 + y^2}$ around the point (0, 0) to gain some sense of whether or not the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2 + y^2}$ exists.

a) Compute the limits $\lim\limits_{\substack{(x,y)\to(0,0)\\ along\ x=0}} \dfrac{2xy}{x^2 + y^2}$, $\lim\limits_{\substack{(x,y)\to(0,0)\\ along\ y=0}} \dfrac{2xy}{x^2 + y^2}$, and $\lim\limits_{\substack{(x,y)\to(0,0)\\ along\ y=x}} \dfrac{2xy}{x^2 + y^2}$.

b) What can you conclude from your answers to part a)? Do your computations "jive" with what you expected after viewing the graph of $z = \dfrac{2xy}{x^2 + y^2}$?

Exercise 2. Consider the function $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$.

a) Examine the limit $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ by computing $\lim\limits_{\substack{(x,y)\to(0,0)\\ along\ y=mx}} \dfrac{x^2 y}{x^4 + y^2}$.

b) Next examine the limit $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ by computing $\lim\limits_{\substack{(x,y)\to(0,0)\\ along\ y=mx^2}} \dfrac{x^2 y}{x^4 + y^2}$.

c) *True or False:* If the limit $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exists along all straight lines $y = mx$ into the origin and the limit is the same along each of these lines, then $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exists. Explain your reasoning.

[1] Feel free to use *Maple* to check your work though!

Adapted from a lab created by Prof. Judy Holdener.

Exercise 3. Most of the limits you have investigated thus far have been taken at the origin, but there is nothing special about the point (0, 0). Most texts and instructors choose this point for the sake of convenience. Since you probably don't want this project to be driven by convenience, let's consider a limit taken at a point other than the origin:

a) Examine the limit $\lim_{(x,y)\to(0,1)} \frac{2x^2}{x^2+y-1}$ by computing the limit along lines coming into (0, 1).

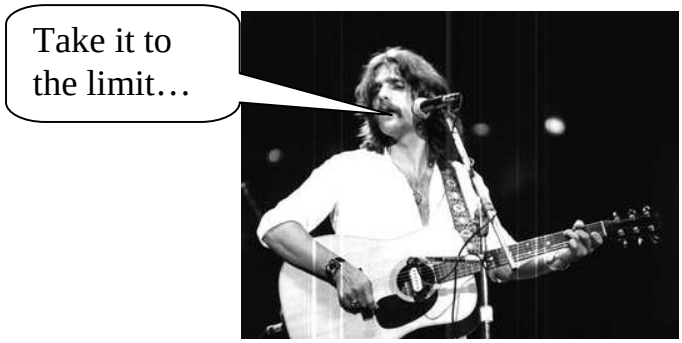
b) Next examine the limit $\lim_{(x,y)\to(0,1)} \frac{2x^2}{x^2+y-1}$ along parabolic paths coming into the point (0, 1).

c) Does the limit $\lim_{(x,y)\to(0,1)} \frac{2x^2}{x^2+y-1}$ exist? Explain.

Exercise 4. For each of parts a) through c) below, determine whether or not the limit exists. If you determine that the limit does not exist, then provide a careful proof of your claim. If you determine that the limit does exist, then explain the "intuitive reasoning" behind your decision, providing an appropriate graph if you think it is helpful.

a) $\lim_{(x,y)\to(1,1)} \frac{x+y-2}{x-y}$ *b)* $\lim_{(x,y)\to(2,-2)} e^{-x-y}$ *c)* $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{x^2+y^2}$

Exercise 5. Up until now, you have only been asked to be rigorous about proving the *nonexistence* of a limit. (Illustrating that two different paths into a point (*a, b*) yield two different limit values of a function *f*(*x, y*) is a rigorous proof that the limit $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.) For the final exercise of this project, let's get a glimpse of the rigor involved in proving the *existence* of a limit. Use an epsilon-delta argument similar to the example done in class to prove that $\lim_{(x,y)\to(0,0)} \frac{x+y}{2+\cos x} = 0$.



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