Separable Differential Equations
Math 112
Professor Smith

1. The Blob
(From *Calculus* by Ostebee and Zorn, Vol 1. page 264)

A certain menacing biological culture (aka "The Blob") grows at a rate proportional to its size. When it arrived unnoticed one Wednesday noon in Chicago's loop, it weighed just one gram. By the 4 p.m. rush hour it weighed 4 grams. The Blob has its "eye" on the Sears Tower, a tasty morsel weighing around 3×10^{12} grams. As soon as The Blob weighs 1000 times as much (i.e. 3×10^{15} grams), it intends to eat the Sears Tower. By what time must the Blob be stopped? Will Fridays rush hour be delayed?

2. Newton's Law of Cooling
(From *Calculus* by Ostebee and Zorn, Vol 1. page 255)

It's a winter day in Frostbite Falls, Minnesota: 10 degrees below zero Celsius. Boris and Natasha stop at a convenience store for hot (initially 90° C) coffee to warm them on the cold, windy walk home. Two types of cups are available, environmentally destructive foam and politically correct cardboard.

Boris wants foam. "What do I care about the greenhouse effect?" he asks. "Besides, remember the proportionality constant k in Newton's law of cooling,

$$y' = k(y - C),$$

where C is the ambient temperature, y is the Celsius temperature of the coffee at time t, and t is the time measured in minutes? Well, for foam, $k = -0.05$. For cardboard, k is a pathetic -0.08."

"Do what you want, Boris," says Natasha. "I'd rather save the world. I'm having cardboard. And make mine decaf."

(a) How long does Boris's coffee stay above 70 degrees? How about Natasha's? How hot is each cup after 5 minutes?
(b) Redo part (a) assuming that Boris and Natasha drink their coffee in the overheated (25° C) convenience store.
(c) What value of k is needed to assure that the coffee, starting at 90° C, is still 70° C after 5 minutes outdoors?

3. Carbon Dating
(From Calculus in Context by Callahan et al., pg. 228)

Virtually all living things take up carbon as they grow. This carbon comes in two principal forms: Normal stable carbon (C12) and radioactive carbon (C14). C14 decays at a rate proportional to the amount of C14 remaining. While the organism is alive, this lost C14 is continually replenished. After the organism dies, through, C14 is no longer replaced, so the percentage of C14 decreases exponentially over time. It is found that after 5730 years, half of the original C14 remains. If an archaeologist finds a bone with only 20% of the original C14 present, how old is the bone?