Recognizing Riemann Sums
Math 112
Professor Smith

For each expression given below, complete the following.

(a) Recognize the limit of the sum as a definite integral. Explicitly state the definite integral as well as what you have chosen for $\Delta x, \bar{x}_k$, and $f(x)$.

(b) State whether the sum is a left, right, or midpoint sum.

(c) Compute the limit of the sum by evaluating the definite integral by hand.

1. $\displaystyle \lim_{n\to\infty} \sum_{j=1}^{n} \frac{2}{n}\left(5+\frac{2j}{n}\right)^{10}$

2. $\displaystyle \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$

3. $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \cos\left(\frac{4(2k-1)}{n}\right)\frac{8}{n}$

4. $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(\frac{k-1}{n}\right)\left(1-\left(\frac{k-1}{n}\right)^{5}\right)$

As an interesting side note, the Riemann sum that appears in question 4 above was chosen because it was an integral part (pun intended!) of a interesting statistics problem. The statistics problem involved devising a method (or more specifically, an unbiased estimator) that could be used to estimate the number of skittles in a jar. It has also proven useful in, for example, estimating the number of tanks that an enemy army possesses. The calculation required relies on recognizing Riemann sums! Pretty cool, huh? If you think that using limited information to estimate the number of skittles in a jar or tanks in an army sounds interesting, then I encourage you to take the Probability Math Stat sequence (Math 336-436).