Homework: Center of Mass

Calculus II, Math 112
Professor Smith

Name:_____

Point Masses

1. The masses m_i are located at the points P_i. Find the moments M_x and M_y and the center of mass of the system $(\bar{x}, \bar{y})$.
$m_1 = 6, m_2 = 5, m_3 = 1, m_4 = 4$.
$P_1 = (1, -2), P_2 = (3, 4), P_3 = (-3, -7), P_4 = (6, -1)$.

Regions in the Plane

We derived the following formulas for the centroid of a region in the plane:

Moment about the y-axis: $M_y = \rho \int_a^b x f(x) dx$

Moment about the x-axis: $M_x = \rho \int_a^b \frac{1}{2}(f(x))^2 dx$

Total mass: $m = \rho \int_a^b f(x) dx$

Centroid $(\bar{x}, \bar{y})$: $\bar{x} = \frac{M_y}{m}, \ \bar{y} = \frac{M_x}{m}$.

A similar argument leads to the following formulas when the region is bounded above by $f(x)$ and below by $g(x)$:

Moment about the y-axis: $M_y = \rho \int_a^b x(f(x) - g(x)) dx$

Moment about the x-axis: $M_x = \rho \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx$

Total mass: $m = \rho \int_a^b f(x) - g(x) dx$

Centroid $(\bar{x}, \bar{y})$: $\bar{x} = \frac{M_y}{m}, \ \bar{y} = \frac{M_x}{m}$.

2. Sketch the region bounded by the curves, and visually estimate the location of the centroid. Then find the exact coordinates of the centroid. You may use Maple to compute the integrals for these problems.

(a) $y = \sqrt{x}, y = 0, x = 9$.

(b) $y = \frac{1}{x}, y = 0, x = 1, x = 2$.

(c) $y = \sin(x), y = \cos(x), x = 0, x = \frac{\pi}{4}$.

(d) $y = x, y = \frac{1}{x}, y = 0, x = 2$.