Power Series Problems

1. Find the radius of convergence of the series

$$\sum_{k=0}^{\infty} \frac{(-3)^k x^k}{\sqrt{k+1}}.$$

2. Find the Taylor series of $f(x) = \sin(x)$ at $x = \frac{\pi}{6}$.

3. Approximate $\int_0^{0.5} x^2 e^{-x^2} dx$ with an error less than 0.001.

4. For each of the following, find the sum of the series.

   (a) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k}}{k!}$

   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{6^{2k}(2k)!}$

   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{5^k k!}$

   (d) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$

   (e) $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$