An Introduction to Maple

This lab is adapted from a lab created by Bob Milnikel.

Some quick tips for getting started with Maple:

[Even before we start, take note of the distinction between Text mode and Math Mode. I'm typing a lot of these instructions in text mode, but unless you're going to be writing papers (or instructive guides to Maple) in Maple, you can stay in Math mode all the time. If you ever try to type an expression or evaluate a function and nothing happens, check to see if you're in Text mode. You can go back and forth between text mode and math mode by using CTRL+T and CTRL+M.]

1) Entering Expressions

To get the expression
$x^2 + 7\,x - 4,$

You can type the 'x' (of course), then type the ^ symbol above the 6 to put you into the exponent and type a 2. However, if you then try to just continue typing 7x-4, you'll get:

> $x^{2 + 7\,x - 4}$

which is not what you're looking for. After typing the 2, hit the right arrow key. This will get you out of the exponent and back onto the main line of the formula. **Type in $x^2 + 7\,x - 4$ below.**

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Fractions are also pretty easy with Maple. To get the expression

$\dfrac{x^3}{x^2 + 1},$

you can type x ^ 3 [rightarrow] as usual, then hit the / key to indicate that you want a fraction. Then go ahead and type in x^2[rightarrow]+1, followed by another [rightarrow] to get you out of the fraction altogether.

Again, there's a potential for a little difficulty with an expression like: $\dfrac{x + 3}{x^2 + 1}$

If you just type x + 3 / x^2[rightarrow]+1[rightarrow] you'll wind up with

> $x + \dfrac{3}{x^2 + 1}$

which is a perfectly nice expression, but not what we're looking for.
To get the intended expression, you need to use parentheses. Type (x+3)/x^2[rightarrow]+1[rightarrow] and you'll get what you're looking for. **Type in $\dfrac{x + 3}{x^2 + 1}$ below.**

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Now that you know how to enter things like polynomials in Maple, you might be interested to know that Maple is really good at algebra. If I type:

> $\left(x^2 + 3\,x - 28\right) \cdot \left(x^2 + 2\,x + 5\right)$

(By the way, I got the *necessary* dot-like multiplication symbol by typing shift-8 to get a *.) I can right-click on the part in blue and have options like "expand".

Try using the expand option. Then right click on the result and use the factor option.

We'll also need to enter things like trigonometric functions, exponential functions, and so forth. Trig functions are pretty easy. You get the sine function by just typing sin(x)

> $\sin(x)$

Similarly for cos, tan, etc. But beware: Maple is finicky about your always putting in the parentheses. sin x doesn't work, nor does sin (x) with a space between the sin and the (x). You get π by typing Pi. (It needs to be capitalized exactly that way. Lowercase gives you the letter, but used as a Greek-letter variable, not the constant that is approximately 3.14159.) **Try typing sin(pi) and sin(Pi).** Notice that the answer is different!

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Another way to get the sine function is to go the menu on the left labeled "Expression". You see trig functions there, as well as exponentials, logs, fractions, roots, some things we haven't talked about but will, and some things that you'd see in Calc II or Calc III. Everything we'll use in this course is either (a) in that menu, (b) can be typed directly pretty easily, or (c) can be accomplished by entering an expression or equation and right-clicking on it.

The Expressions menu is great for square roots, fourth roots, etc., although you can always use fractional exponents as well. (Remember that x^(1/2) is the square root, x^(1/4) is the fourth root, etc.)

Some things that look easy to type are actually misunderstood if you type them into Maple in the obvious way. Two examples are e^x and |x|.

For the base-e exponential function, you have two options. (1) You can go to the Expression menu on the left and hit the e^a button. You'll have an "a" there that you can replace by x or whatever else you want to put in the exponent. (2) You can type

> $\exp(x)$

which you see Maple interprets as e^x. You can even combine functions like:

> $\exp(\sin(x))$

and Maple is fine with that.

However, if you type literally the letter e, the ^ symbol to get yourself into the exponent, and the letter x, Maple will interpret that e as a variable you haven't defined yet, not as the constant that is approximately 2.71828.

The other odd one is the absolute value. Just as with the exponential function, Maple will be confused if you just type a vertical bar, then an expression, then another vertical bar. It *looks* like the absolute value function, but Maple won't interpret it that way. You need to use the absoute value button from the Expression menu or you can use the typed-out

> $\text{abs}(x^3)$

in its place. As you can see, Maple interprets abs(whatever) as the absolute value of whatever.

2) Calculus operations on symbolic expressions

Maple knows how to take limits and how to differentiate and integrate. You can use the symbols in the Expression menu or you can use the limit, diff, and int commands, as shown in these next examples.

> $limit(\sin(x), x = \text{Pi})$

The above command computes the limit of sin*(x)* as *x* approaches π.

Try computing the limit of $\dfrac{\sin(x)}{x}$ **as *x* approaches *0*.**

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Next are examples of differentiation and antidifferentiation.

> $diff(\sin(x), x)$

> $int(\sin(x), x)$

Note that you have to tell Maple both the function that you want to differentiate or integrate as well as the variable with respect to which you are performing the operation. If you leave out the ',x' you will get an error, as in the following example:

> $diff(\sin(x))$

Error, invalid input: diff expects 2 or more arguments, but received 1

The error message tells you that you have to give Maple another piece of information (the ',x') before it knows what to do. This should not be surprising because depending on which variable you differentiate with respect to, the answer can be different. i.e. the derivative of *sin(x)* with respect to *x* is *cos(x)*, but the derivative of *sin(x)* with respect to *w* is 0 because there are no *w*'s in *sin(x)*. So Maple needs both the function and the variable you're differentiating or integrating with respect to.

You can also ask Maple to differentiate a function repeatedly. For example to the find the 10th derivative of x^{11}, you would type:

> $diff(x^{11}, x\$10)$

Try getting Maple to compute the derivative of $\cos(e^x)$ with respect to x.

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Now get Maple to compute the 7th derivative of $\cos(e^x)$ with respect to x.

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You can also compute the definite integral in the following way.

> $int(x^2, x = 3..5)$

This gives you the definite integral of x^2 with respect to *x* over the interval [3,5], i.e. $\int_3^5 x^2 \, dx$.

2) Plotting Functions

Maple is great at plotting functions. Here are a few examples.

> $plot(\sqrt{x})$

Notice a few things. First of all, the word "plot" is all lowercase, and it needs to be. Secondly, notice the parentheses following the word. They have to come *immediately* after the word "plot", and they always have to be there. Any information about what you want plotted and how has to come inside those parentheses. Finally, notice the default x- and y-windows. The default x-window is -10 to 10; the default y-window is whatever the range is for the function restricted to inputs between -10 and 10. We can change several things about this plot:

> $plot(\sqrt{x}, x = 0..16, y = -8..8, color = violet, thickness = 2, caption = $ "A Purple Curve"$)$

It's worth noting two things here. First of all, the way to indicate the interval from 0 to 16 is with *two* dots between the 0 and the 16: 0..16. Secondly, I'll point out that you aren't allowed to set y-values for the window unless you set x-values as well. But if you want to set only x-values, that's fine.

> $plot(\sqrt{x}, x = 0..16)$

Finally, it's sort of cool -- and soon will be quite useful to us -- to be able to plot multiple functions in

one window on one set of axes.

> $plot\left(\left[\sqrt{x}, \sin(x), \exp\left(-x^2\right)\right], x=-2..2, y=-2..2, color=[blue, orange, violet], thickness=2\right)$

Try plotting x^2, $\tan(x)$, and $-2\,x$ together on the interval $\left[-\dfrac{1}{2}\,\pi, \dfrac{1}{2}\,\pi\right]$, with y-values in $[-2, 2]$.

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3) Defining Functions

For reasons that will soon become clear (I hope) it's sometimes useful to give a function a definition, rather than just always typing out its full formula. For example if we wanted to assign the name "f" to the function

> $\dfrac{1}{\sqrt{2 \cdot Pi}}\exp\left(-\dfrac{x^2}{2}\right)$

we would type the following: The letter f, a colon (:), an equals sign (=), the letter x, a dash (-), a greater-than symbol (>) and then the expression could be entered as normal. To repeat, in text that looks like:

f := x-> [The expression] But if we do it in Math mode, we get:

> $f := x \rightarrow \dfrac{1}{\sqrt{2 \cdot Pi}}\exp\left(-\dfrac{x^2}{2}\right)$

Notice how Maple knows that the : and = get combined into that elongated equals sign, and that the - and > get combined into the arrow. The := is our way of telling Maple "Hey, I'm about to define something." When preceded by an "f", we are saying, "Let f be defined to be the following..."

The arrow is actually a really good reminder that we're defining a *function*. What a function does is take x-values to output values by some rule. The arrow says "send x to...".

With the function defined, we can do things like evaluate f(3).

> $f(3)$

Of course, that's not very helpful, but we can right-click on it to get an approximation. **Right click on the answer above and use the approximate option to get a 10-digit approximation to $f(3)$.** You can also use the evalf command if you prefer typing a command over right-clicking.

The main thing defining functions is good for for our purposes, though, is graphing.

> $plot(f(x), x=-3..3, y=0..0.5, color=blue, thickness=2)$

Note that if you try to plot f, and not $f(x)$, Maple will give you an error. Since you defined f as a function of x, Maple will only recognize it in that context.

4) Loops

A for loop is a group of commands (that are actually altogether one command) that allows you to execute commands repeatedly. Suppose for example that you wanted to evaluate the function $f(x) = x^2$ at $x=1$, $x=2$, $x=3$, $x=4$, ..., all the way up through $x=10$. You can compute these and store them in the variables A_1 through A_{10} by doing the following.

> $f := x \rightarrow x^2$
> **for** k **from** 1 **to** 10 **do**
> $A[k] := f(k)$
>   **od**

To type in a loop yourself, you need to use SHIFT+ENTER to get from each line to the next. If you hit enter before typing in the entire command (beginning with for and ending with od), Maple will give

you an error.

Now suppose that you wanted to evaluate $f(x) = x^2$ at $x=1$ through $x=100$. You want to store all of the values (so that you could then use them and/or plot them), but you probably don't want to see them all. To get Maple to do the computation but suppress the output, use a colon at the end of the command (after od). Execute the following loop.

> **for** k **from** 1 **to** 100 **do**
   $A[k] := f(k)$
   od:

Now try executing the following line, which asks Maple to output the value of A_{30}.

> $A[30]$

Note that Maple has computed all of the values of A_1 through A_{100}, so it knows them and can output them, but it doesn't show you all of them when it computes them (thankfully!) since you asked it to hide the output by putting a colon at the end of the command.

Now let's write a loop that will compute the sum 2+4+6+8+10+...+40. We'll start by asking Maple to find 2, then 2+4, then 2+4+6, then 2+4+6+8, and so on until we make it all the way up to 40. As we go, we'll want to store the current sum in a variable, so let's make a variable S, which will be 0 to start. At each step, we need to add on a different number, but the number to be added on follows a nice pattern, first it's 2, then 4, then 6, etc. In other words the number to be added on at each step is precisely 2 times the index k. Now we're almost ready to write the loop. The last thing to be aware of before we do is that if you are going to have more than one line in the interior of a for loop, it's important that you end each line with a semicolon so that Maple knows when it's done with one command and can move on to the next command. Okay, here's what the loop should look like.

> $S := 0;$

> **for** k **from** 1 **to** 20 **do**
   $NumberToBeAdded := 2 \cdot k;$
   $S := S + NumberToBeAdded;$
   od;

Several important points:
1. You can name variables almost anything in Maple (there are a few names that are reserved -- for instance, you can't use exp for anything but the exponential function). So naming a variable *NumberToBeAdded*, while a long name, is an informative name, and you'll know what it's being used for.
2. The line *S:=S+NumberToBeAdded* can be somewhat confusing, but it is a common type of step in a for loop. Each time Maple comes to that line, it begins by computing whatever is on the right side (so in this example, it takes whatever *S* currently is and adds *2k* to it). THEN, it assigns that number to the variable *S*. So what happens in, for example, the step when *k=1*, is *S* has a starting value (in this case, *S=0*), then Maple computes *S+NumberToBeAdded*, which is *0+2=2,* then Maple assigns that number to *S*. So at the beginning of the step *S* had a value of *0*, at the end of the step *S* had a value of *2*. The next time around *S* will start with a value of *2* and end with a value of *6*, and so forth.
3. The ending value of *k* is *20* because we want to stop adding things on when we add on *40*. Since *40= 2*20*, we want to stop when *k=20*.
4. I made this loop with an extra step to illustrate a couple points but it could be streamlined as follows.

> $S := 0;$
   for k **from** 1 **to** 20 **do**
   $S := S + 2 \cdot k;$
   od;

Note that it's important when we do this second loop that we reset *S* to be *0* because otherwise we'll start with a sum of *420* (because Maple remembers that the current value of *S* is *420*) and then keep adding on more.

5) Further Exercises

1. Get Maple to give you the cosine of $\dfrac{3}{4}\pi$, both as an exact answer and as an approximation with 20 digits.

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2. Input the following expression: $\dfrac{\dfrac{1}{x}+\dfrac{1}{x-3}}{\dfrac{x}{x-3}+x^2}$. Then get Maple to simplify it for you.

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3. Try plotting $\sin\left(x^2\right)$ from x equals 0 to 100.

Why do we get such a 'bad' graph?

Change the domain until you get a better graph. At what point does the graph seem to get out of hand?

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4. What happens if you type $v = 1$ followed by $v + 2$?

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What happens if you type $v := 1$ followed by $v + 2$?

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Can you explain the difference in the result?

5. Find log base 5 of 20 as a decimal approximation with 5 digits.

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Find the natural log of 20 as a decimal approximation with 5 digits.

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6. Get Maple to compute each of the following: $\sin\left(\arcsin\left(\dfrac{\sqrt{2}}{2}\right)\right)$, $\arcsin\left(\sin\left(\dfrac{\pi}{4}\right)\right)$, $\arcsin\left(\sin\left(\dfrac{5\pi}{6}\right)\right)$.

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Explain why the last answer is not $\dfrac{5\pi}{6}$.

7. Find the 20th antiderivative of $x^2 \sin(x)$.

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8. Compute $\int_0^3 x^4 e^{3x} \, dx$.

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9. Compute the limit of $\dfrac{\sin(x+h) - \sin(x)}{h}$ as h approaches 0. What is the answer and what does it represent?

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10. Write a loop that computes the sum 1+3+5+7+...+31.

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11. It's a good idea to familiarize yourself with Maple's excellent help menu. Start by typing *help(plot)* in the command prompt below. This will take you to a page with a description of the command and all of its options as well as a variety of examples. The best way to use the help menu is often to parues the examples and then modify them for your needs. Copy and paste one example that you find to be fun into this page and execute it. Might I suggest checking out the *plot3d* command for exceptionally cool plots?

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