Final ODK Study Guide

This is intended to be a list of suggested topics, definitions, theorems, and techniques for study. It is not intended to be a comprehensive list of all possible exam questions.

1. Euler's method
    - Know the formula and be able to apply it to an initial value problem (using positive or negative Δt).
2. The Fundamental Theorem of Calculus
    - Know both parts of the theorem.
    - Be able to apply the theorem in problems with a given function or with a graph.
3. Riemann sums
    - Be able to compute a left endpoint, right endpoint, midpoint, or trapezoidal rule Riemann sum.
    - Understand the definitions and concepts: Riemann sum, integrability, definite integral.
    - If given the error bound formulas, be able to compute a bound on the error from a Riemann sum approximation of an integral.
4. Integration techniques
    - Given a definite or indefinite integral, be able to choose the appropriate technique and apply it correctly.
    - Techniques: u-substitution, integration by parts, partial fractions.
    - Be able to rewrite or simplify the integral first, if necessary. Also know when partial fractions is applicable and know the general form of a partial fraction decomposition.
5. Applications of integration
    - Area between curves – be able to set up and compute the area of a region in the plane with respect to both x and y. Be able to draw a representative rectangle.

- Volumes by slicing/revolution – Be able to set up and compute the volume of a solid with respect to x or y, as appropriate. Be able to draw a representative rectangle and disk or washer.

6. Separable differential equations

   - Be able to recognize separable and inseparable differential equations.
   - Be able to apply the technique of separation of variables to solve a differential equation or initial value problem.
   - Be able to derive the solution to the equation for exponential growth.
   - Be able to answer questions with your solution to an initial value problem.

7. Taylor polynomials

   - Know the formula.
   - Understand the relationship between Taylor polynomials and polynomials.
   - If given the formula for the error bound, be able to use it to find an error bound for a Taylor polynomial approximation.
   - Be able to compute the Taylor polynomial for a function centered at a.

8. Improper integrals

   - Recognize if/how an integral is improper.
   - Be able to use limits appropriately to compute an improper integral.
   - Be able to use the Comparison Test for nonnegative improper integrals.

9. Sequences

   - Know the definition.
   - Be able to go from a list to a rule and vice versa (for both recursively defined sequences and non-recursive sequences).
   - Be able to appropriately apply L'Hopital's rule to find the limit of a sequence (by defining an associated differentiable function first!)
   - Know and be able to apply the Sandwich Theorem and the Monotone Convergence Theorem.
   - Know the limits of important sequences (such as $\left(1+\frac{x}{n}\right)^n$) and be able to apply rules as necessary (sum rule, etc).

10. Series

    - Definitions and concepts: sequence of terms, sequence of partial sums, sigma notation, series, convergence/divergence, geometric series, p-series, sum and constant multiple rules, telescoping series, n^{th} term test *for divergence*, absolute convergence / conditional convergence / divergence.
    - Be able to go from a sum to sigma notation and vice versa.
    - Be able to derive the formula for the n^{th} term in the sequence of partial sums for a geometric series.
    - Be able to give a detailed, graphical explanation of the bounds given by the integral test theorem (from which the theorem itself follows).
    - Be able to find a rational expression for a repeating decimal.
    - Understand the relationship between $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=n}^{\infty} a_k$ for some integer n.

- Tests for nonnegative series: Know and be able to show that all hypotheses are satisfied, be able to choose the appropriate test to apply, be able to apply each test correctly to show convergence or divergence, be able to use the Direct Comparison Test, Integral Test, and Alternating Series Test to approximate and/or find bounds for the sum of a series. If necessary, use algebra to rewrite a series before showing convergence/divergence.
  - Direct Comparison Test
  - Limit Comparison Test
  - Integral Test
  - Ratio Test
- Know the relationship between absolute convergence and convergence.
- Alternating Series Test.
- Be able to compute the sum of a geometric or telescoping series.

11. Power series

- Know the definitions.
- Know and be able to use the theorems pertaining to convergence of a power series (as an interval) and to differentiation and integration of power series (including what you know about the interval of convergence).
- Be able to derive the formulas for the series for $\arctan(x)$, $\ln(1+x)$, $\frac{1}{1+x^2}$, etc.
- Be able to multiply series.
- Be able to find the interval of convergence (including endpoint behavior!) and radius of convergence of a power series.
- Be able to use substitution and multiplication by functions of x to create new series from known series. Know how these operations affect the interval of convergence.
- Be able to compute nonelementary integrals using power series represenations.

12. Taylor series

    - Know the definition.
    - Know the relationship between power series and Taylor series. i.e. Given a function f, does it have a power series about $x = a$? If it does, is it the Taylor series about $x = a$? Does that series have to converge? Does it have the same domain as f? Will it converge to f? Everywhere?
    - Be able to compute the Taylor series about $x = a$ for a function f.
    - Be able to use Taylor's Theorem to show that the Taylor series for a function converges to the function.

13. Important series to **know**:

    - Maclaurin series for $\sin(x), \cos(x), \frac{1}{1-x}, e^x$.
    - Given that you know these and understand multiplication/substitution, be able to recognize a series as a given function. You may need to use this to compute the sum of a series.

14. Be able to give examples of important objects, i.e. a monotone sequence that converges/diverges, a conditionally convergent series, a geometric series, a p-series, ...