Homework: Arc Length and Intro to Volumes Calculus II, Math 112

Name:_____

Arc Length

1. Find the arc length of the curve for each of the following. Compute each integral by hand!

(a)
$$y = \frac{1}{3}(x^2 + 2x)^{3/2}$$
 between $x = 0$ and $x = 1$.
(b) $y = \frac{x^4}{4} + \frac{1}{8x^2}$ between the points $(1, \frac{3}{8})$ and $(3, \frac{1459}{72})$.
(c) $y = \ln(\sec(x))$ over the interval $\left[0, \frac{\pi}{4}\right]$.

Volumes of Solids

You may use Maple or a calculator to compute the integrals once you've set them up.

2. Let R be the region between the graph of the function and the x-axis on the given interval. Find the volume V of the solid obtained by revolving R about the x-axis.

- (a) $f(x) = 1 + x^2$ on [-1, 2].
- (b) $f(x) = x(x^3 + 1)^{\frac{1}{4}}$ on [1, 2].

3. Find the volume of the solid generated by revolving about the x-axis the region between the graphs of the given equations

- (a) $y = \frac{1}{2}x^2 + 3$ and $y = 12 \frac{1}{2}x^2$.
- (b) y = 5x and $y = x^2 + 2x + 2$.

4. Find the volume of the solid generated by revolving the region between the graphs of the equations $y^2 = x$ and x = 2y about the y-axis.

5. Find the volume V of the solid with the given information about its cross sections.

- (a) The base of the solid is an isoceles right triangle whose legs are each 4 units long. The cross sections parallel to one of the legs are semicircular.
- (b) The solid has a circular base with radius 1, and the cross sections perpedicular to a fixed diameter of the base are squares. (Hint: center the base at the origin.)

⁰Parts of this worksheet are adapted from a worksheet created by Carol Schumacher.