Volumes by Parallel Cross Section

1. Let R be the region between the graph of the function and the x-axis on the given interval. Find the volume V of the solid obtained by revolving R about the x-axis.

   (a) $f(x) = 1 + x^2$ on $[-1, 2]$.

   (b) $f(x) = x(x^3 + 1)^{\frac{1}{4}}$ on $[1, 2]$.

2. Find the volume of the solid generated by revolving about the x-axis the region between the graphs of the given equations

   (a) $y = \frac{1}{2}x^2 + 3$ and $y = 12 - \frac{1}{2}x^2$.

   (b) $y = 5x$ and $y = x^2 + 2x + 2$.

3. Find the volume of the solid generated by revolving the region between the graphs of the equations about the given axis.

   (a) $y = x$ and $y = \sqrt{x}$ about $y = 1$.

   (b) $y = x$ and $y = \sqrt{x}$ about $y = -2$.

   (c) $y^2 = x$ and $x = 2y$ about the $y-$axis.

   (d) $y = x$ and $y = \sqrt{x}$ about $x = 2$.

4. Find the volume V of the solid with the given information about its cross sections.

   (a) The base of the solid is an isoceles right triangle whose legs are each 4 units long. The cross sections parallel to one of the legs are semicircular.

   (b) The solid has a circular base with radius 1, and the cross sections perpedicular to a fixed diameter of the base are squares. (Hint: center the base at the origin.)

5. Derive formulas for the volumes of the following solids:

   (a) A right circular cone with height h and radius (of the base) r.

   (b) The "cap" of a sphere resulting from slicing a sphere of radius r at a distance h from its center.

   (c) A right pyramid with a square base of side length a and height h.

---

[0]This worksheet was adapted from a worksheet created by Carol Schumacher.