Some Series Practice

For each of the following, determine if the series converges or diverges. State which test you're using and show that all hypotheses are satisfied.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 + 1}{k^2\sqrt{k}}$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 + 1}{k^3\sqrt{k}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n(n + 2)2^n}{3^n}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{4n + 25}{5n + 10}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1}$

6. $\displaystyle\sum_{j=1}^{\infty} \frac{1}{j^3}$ (no p-series knowledge allowed for this one).

7. $\displaystyle\sum_{k=0}^{\infty} \frac{5k + 10}{25k^2 + 54k + 87}$

8. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{1000^n}$

9. $\displaystyle\sum_{m=1}^{\infty} \frac{(-1)^n 4^n (n + 2)}{(n + 1)!2^n}$

10. $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k + 5}$

11. For question 6, find an upper and lower bound on the sum.

12. For question 10, find an upper and lower bound on the sum.

13. For question 5, bound the magnitude of the error on the approximation S_5.