Math 112 Series Comments

Several students turned in incorrect solutions to the following problem and it's important that everyone understand the issues. Determine convergence of the series $\sum_{n=0}^{\infty} \frac{(-43)^{n+1}}{n!}$.

You have two options for dealing with this series.

1. Ratio test for *absolute* convergence. You cannot apply the ratio test immediately because there are negative terms. You can apply it to the absolute value of the terms and hope that the series converges absolutely.

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n\to\infty} \left| \frac{(-43)^{n+2}}{(n+1)!} \cdot \frac{n!}{(-43)^{n+1}} \right| && (1) \\
&= \lim_{n\to\infty} \left| \frac{-43}{n+1} \right| && (2) \\
&= \lim_{n\to\infty} \frac{43}{n+1} && (3) \\
&= 0. && (4)
\end{aligned}$$

   Since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 < 1$, the series converges absolutely by the ratio test. Therefore, the series converges.

2. Rewrite the series and treat it as an alternating series. It is not written as $(-1)^{n+1} c_n$ already! Rewrite as

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 43^{n+1}}{n!}.$$

   Then $c_n = \frac{43^{n+1}}{n!}$. *Now*, we can apply the alternating series test, because *now* we have an alternating series.
   Alternating series conditions:

   - $c_n = \frac{43^{n+1}}{n!} \geq 0$ for all n.

- $c_n \geq c_{n+1}$? For this to be true, we would need

$$\begin{aligned} \frac{43^{n+1}}{n!} &\geq \frac{43^{n+2}}{(n+1)!}, \text{ which is equivalent to} && (5) \\ \frac{43^{n+1}}{43^{n+2}} &\geq \frac{n!}{(n+1)!} && (6) \\ \frac{1}{43} &\geq \frac{1}{n+1} && (7) \\ n+1 &\geq 43. && (8) \end{aligned}$$

  This is true for $n \geq 42$. So $c_n \geq c_{n+1}$ for all n greater than or equal to 42.

- $\lim_{n\to\infty} c_n = 0$? This is true but requires a fairly sophisticated argument. The sequence is not a decreasing sequence (in fact, it gets as large as 1.07×10^{19}), but it does eventually go to zero. If you made it this far on an exam, you would receive most, but not all, of the points for the problem. The best tactic for this problem is use the ratio test for absolute convergence.