

## Fermentation<sup>1</sup>

Wine is made by yeast; yeast digests the sugars in grape juice and produces alcohol as a waste product. This process is called fermentation. The alcohol is toxic to the yeast, though, and the yeast is eventually killed by the alcohol. This stops fermentation, and the liquid has become wine, with about 8%-12% alcohol content. Although alcohol isn't a "species," it acts like a predator on yeast. In this lab, you will discuss a model that describes the interactions between sugar, yeast and alcohol.

**Constructing the First Model:** Suppose that  $Y$  is the amount of yeast that is present and  $A$  is the amount of alcohol present in the mixture. These are measured in pounds.

**Question 1:** In the absence of alcohol, the yeast grows logistically. That is, if yeast had the brains to produce a good plumbing system, it would blissfully grow to the capacity of its environment. Unfortunately, yeasties are stupid as doornails, so when they produce alcohol as a waste product, they just leave it lying around willy nilly to poison them. Since the poisoning process requires interaction between yeast and alcohol, the yeast cells die at a rate proportional to the product of the amount of yeast and the amount of alcohol present. Moreover, the yeast consume the sugar in the grape juice which limits their food supply. Assume that the carrying capacity of the environment is 10. Write a differential equation that describes the rate of change in the amount of yeast as a function of  $Y$  and  $A$ .

**Question 2:** The alcohol is produced by the yeast, it is therefore reasonable to assume that the amount of alcohol produced is proportional to the amount of yeast that is present. (You should be sure you understand why this is so.) Suppose that each pound of yeast produces .05 lbs of alcohol per hour. Write a differential equation that represents the rate of change of the  $A$  as a function of  $Y$ .

You should now have a system of two differential equations that model the interactions between  $Y$  and  $A$ . For now, you will assume that the sugar is not used up by the yeast and that the sugar level is kept at a threshold that leaves the carrying capacity at 10. At the beginning of the wine making process, .5 lbs of yeast is dumped into a vat of grape juice. Assume that the vat contains 25 lbs of sugar at the outset. The yeast feeds on the sugar in the juice and begins to grow; this starts the fermentation process.

**Analysis of the First Model:** The unknown coefficients in your differential equation for  $Y$  represent, respectively, the growth rate of the yeast population in the absence of alcohol and the presence of plenty of sugar and room and the toxicity rate of the alcohol. (*Which is which?*) For now, assume that the natural growth rate is .2 and the toxicity rate is .1

**Question 3:** Analyze the model using Euler's method in order to produce graphs that show the amount of yeast and alcohol over time. Also give the final amounts of yeast and alcohol. Then describe in words what happens. How long does it take for the process to end? (Define this to be the time when

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<sup>1</sup> This lab is adapted from the book Calculus in Context by Callahan et. al. Some portions of the lab are taken verbatim from that source.

there is less than .01 lb of yeast left.) How much alcohol do you end up with? How well do your results fit your intuition about how the fermentation process ought to proceed?

**Question 5---Varying parameters:** What happens to the wine making process if the rates of toxicity and alcohol production vary? (How long does it take for the process to end? What does it do to the overall alcohol content of the wine?)

- a) What happens to the wine making process if you decrease the toxicity rate of the alcohol by a factor of 5?
- b) What happens to the wine making process if you increase the rate of alcohol production by a factor of 5?
- c) What happens if you do both at once? (Decrease toxicity by a factor of 5 and increase alcohol production by the same factor.)
- d) Experiment and describe the long-term effect on the fermentation process of different values for the toxicity and the alcohol production.

**Constructing the Second Model:** In this model you will assume that the sugar is consumed and that the carrying capacity therefore depends on the amount of sugar present. Assume that the amount of sugar is  $S$  and that the carrying capacity is  $.4S$ .  $S$  is also measured in pounds.

**Question 6:** It is reasonable to assume that the sugar is consumed in amounts proportional to the amount of yeast present in the juice. Suppose that the yeast consumes .15 lbs of sugar per lb of yeast. Write a differential equation that describes what happens to the sugar over time.

**Question 7:** Now use Euler's method to analyze your refined model. What happens? How long does it take before there is only .01 lb of yeast left? How much sugar is left? How much alcohol has been produced by that time?