Math 111 – Limits and Derivatives Worksheet

1. Complete problems 27 – 36 in section 2.3.

2. Let $f(x) = \frac{x^3-3x^2-10x}{x^2-4x-5}$ and consider the following limit.

$$\lim_{x\to 5} f(x) = \lim_{x\to 5} \frac{x^3 - 3x^2 - 10x}{x^2 - 4x - 5}$$

   (a) Use Maple to graph the function f(x) in the window -10 ≤ x ≤ 10, -10 ≤ y ≤ 10. Sketch the graph below.

(b) If possible, compute $f(5)$. If necessary, correct your graph above to fit your findings.

(c) Notice that $f(x)$ can be factored as follows.

$$f(x) = \frac{x^3 - 3x^2 - 10x}{x^2 - 4x - 5} = \frac{x(x-5)(x+2)}{(x-5)(x+1)}$$

Explain why $f(x) \neq \frac{x(x+2)}{x+1}$.

(d) Use Maple to graph the function $\frac{x(x+2)}{x+1}$ in the window -10 ≤ x ≤ 10, -10 ≤ y ≤ 10. Sketch the graph below.

(e) Even though $f(x) \neq \frac{x(x+2)}{x+1}$, explain why $\lim_{x\to 5} f(x) = \lim_{x\to 5} \frac{x(x+2)}{x+1}$.

(f) Compute $\lim_{x\to 5} \frac{x^3-3x^2-10x}{x^2-4x-5}$.

3. Let $f(x) = \frac{x^2-9}{x^2+5x-14}$ and consider the following limit.

$$\lim_{x\to 2} f(x) = \lim_{x\to 5} \frac{x^2-9}{x^2+5x-14}$$

(a) Factor $f(x)$. Can you use the method of problem 2 to find this limit?

(b) Use Maple to sketch a graph $f(x)$. From your graph, determine $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$.

(c) What can you say about $\lim_{x\to 2} f(x)$?

4. Let $f(x) = |x-2|$. When we compute the derivative of $f(x)$ at $x = a$, $\mathrm{f}'(\mathrm{a}) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, we compute the limit of the function $\frac{f(a+h)-f(a)}{h}$, whose independent variable is h, as h goes to zero. In this problem, we will use limits to see why the derivative of $f(x) = |x-2|$ does not exist at $x = 2$.

(a) Show that

$$\frac{f(2+h)-f(2)}{h} = \frac{|h|}{h}.$$

(b) Sketch a graph of $\frac{|h|}{h}$. Explain why

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{|h|}{h}$$

does not exist. (The answer, 'it goes to $\frac{0}{0}$,' is *incorrect*!)

(c) Sketch a graph of $f(x)$. Explain the connection between your findings in part (b) and the graph of f(x).

5. Let $f(x) = x^{1/3}$. Repeat the process of question 4 to see that the derivative of $f(x)$ does not exist at $x = 0$.