Math 224
Thursday, November 8, 2007
The Gram-Schmidt Process

How to find an orthonormal basis for a subspace W of $\mathbf{R}^n$

1. Find a basis $\{\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_k}\}$ for W. Often, this will be given.
2. Let
$$\mathbf{v_1} = \mathbf{a_1}$$
3. $$\mathbf{v_2} = \mathbf{a_2} - \frac{\mathbf{a_2} \cdot \mathbf{v_1}}{\mathbf{v_1} \cdot \mathbf{v_1}}\mathbf{v_1}$$
4. $$\mathbf{v_3} = \mathbf{a_3} - \frac{\mathbf{a_3} \cdot \mathbf{v_1}}{\mathbf{v_1} \cdot \mathbf{v_1}}\mathbf{v_1} - \frac{\mathbf{a_3} \cdot \mathbf{v_2}}{\mathbf{v_2} \cdot \mathbf{v_2}}\mathbf{v_2}$$
5. Continue this process, at each stage subtracting from $\mathbf{a_j}$ its projection on the subspace generated by $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_{j-1}}$.
6. Stop when you have k orthogonal vectors:
$$\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}.$$
These form an orthogonal basis for W.
7. You can then normalize the vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}$ to obtain an orthonormal basis
$$\mathbf{q_1}, \mathbf{q_2}, \ldots, \mathbf{q_k}$$
for W.