Math 224
Thursday, October 18, 2007

Theorem 5.2: Matrix Summary of Eigenvalues of A

Let A be an $n \times n$ matrix, and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be (possibly complex) scalars and let $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$ be non-zero vectors in n-space (n-space means either $\mathbf{R}^n$ or $\mathbf{C}^n$). Let C be the $n \times n$ matrix having $\mathbf{v_j}$ as j-th column vector, and let D be the $n \times n$ matrix having the λ_i on the main diagonal and 0's elsewhere.

1. Compute AC (in terms of A and the column vectors $\mathbf{v_i}$).

2. Compute CD (in terms of the column vectors $\mathbf{v_i}$ and the scalars λ_i).

3. Conclude that $AC = CD$ if and only if:

4. Restate the conclusion above in words:

Definition 5.3: An $n \times n$ matrix A is **diagonalizable** if there exists an invertible matrix C such that $C^{-1}AC = D$, where D is a diagonal matrix. We say that the matrix C **diagonalizes** the matrix A.

Corollary 1: A Criterion for Diagonalization

Let A be an $n \times n$ matrix, and let C and D be as in Theorem 5.2. Complete the following statements.

1. The $n \times n$ matrix C is invertible if and only if rank(C)

2. rank(C) (insert answer from previous statement) if and only if the column vectors of C are

3. If (and only if) C is invertible, we can rewrite $AC = CD$ as

4. Conclude: An $n \times n$ matrix A is diagonalizable if and only if there are n eigenvectors of A that are

5. We can restate this conclusion as follows. An $n \times n$ matrix A is diagonalizable if and only if n-space has a basis consisting of

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Corollary 2: Computation of A^k

Suppose that A is an $n \times n$ matrix A with n eigenvectors and eigenvalues. Construct the matrices C and D (as in Theorem 5.2) so that $AC = CD$. Suppose that the n eigenvectors are independent, so that C is invertible, and we have $A = CDC^{-1}$. Find a nice expression for A^k.