Math 224
Homework: Structure of the Solution Set of $A\mathbf{x} = \mathbf{b}$
Solution

Let $A\mathbf{x} = \mathbf{b}$ be a linear system.

1. Let $\mathbf{p}$ be a solution of $A\mathbf{x} = \mathbf{b}$, and let $\mathbf{h}$ be a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$. Show that $\mathbf{p} + \mathbf{h}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

   Solution. $A(\mathbf{p} + \mathbf{h}) = A\mathbf{p} + A\mathbf{h} = \mathbf{b} + \mathbf{0} = \mathbf{b}$

2. Now let $\mathbf{q}$ be any solution of $A\mathbf{x} = \mathbf{b}$. Show that $\mathbf{q} - \mathbf{p}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

   Solution. $A\mathbf{q} - \mathbf{p} = A\mathbf{q} - A\mathbf{p} = \mathbf{b} - \mathbf{b} = \mathbf{0}$

3. Conclude that every solution of $A\mathbf{x} = \mathbf{b}$ can be written in the form $\mathbf{p} + \mathbf{h}$, where $\mathbf{p}$ is a solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{h}$ is a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$

   Solution. From $\mathbf{q} - \mathbf{p} = \mathbf{h}$, it follows that $\mathbf{q} = \mathbf{p} + \mathbf{h}$, as needed.

Congratulations! You just proved Theorem 1.18!