
Math 224

Homework: Structure of the Solution Set of $A\mathbf{x} = \mathbf{b}$

Let $A\mathbf{x} = \mathbf{b}$ be a linear system.

1. Let \mathbf{p} be a solution of $A\mathbf{x} = \mathbf{b}$, and let \mathbf{h} be a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$. Show that $\mathbf{p} + \mathbf{h}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

2. Now let \mathbf{q} be any solution of $A\mathbf{x} = \mathbf{b}$. Show that $\mathbf{q} - \mathbf{p}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

3. Conclude that every solution of $A\mathbf{x} = \mathbf{b}$ can be written in the form $\mathbf{p} + \mathbf{h}$, where \mathbf{p} is a solution of $A\mathbf{x} = \mathbf{b}$ and \mathbf{h} is a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$.

Congratulations! You just proved Theorem 1.18!