Practice Problems: Taylor and Maclaurin Series

1. Find the Taylor series for $f(x) = e^x$ centered at $x = 3$.
2. Find the Maclaurin series for $f(x) = e^{5x}$.
3. Find the Taylor series for $f(x) = \sin x$ centered at $x = \pi/2$.
4. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = \cos(\pi x)$.
5. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = e^{-x/2}$.
6. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = x^2 e^{-x}$.
7. Use a known Maclaurin series to obtain the Maclaurin series for the function $f(x) = \dfrac{\sin x}{x}$. Use the series that you obtain to evaluate the indefinite integral $\displaystyle\int \frac{\sin x}{x}\, dx$ as an infinite series.
8. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$.
9. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$.
10. The graph of $f(x)$ is shown below. Explain why the series
$$1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + \cdots$$
is *not* the Taylor series of f centered at $x = 1$.