Math 224
Quiz 7
Thursday, November 29, 2007

Note: You are allowed to use Maple for this quiz, but you must show all work to receive credit.

1. Is the matrix
$$A = \begin{bmatrix} 2/7 & -3/7 & 6/7 \\ 3/7 & 6/7 & 2/7 \\ -6/7 & 2/7 & 3/7 \end{bmatrix}$$
orthogonal? Why or why not?

2. Let $D = C^{-1}AC$ be a diagonal matrix, where C is an orthogonal matrix. Show that A is symmetric.

3. Find the projection matrix for the subspace

$$W = \text{sp}([1, 2, 1, 1], [-1, 1, 0, -1])$$

in $\mathbb{R}^4$, and use it to find the projection of $\mathbf{b} = [1, 2, 1, 3]$ on W.

4. Show that the projection matrix

$$P = A(A^T A)^{-1} A^T$$

satisfies $P^2 = P$. (Here A is the $n \times k$ matrix with column vectors $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_k}$, where $\{\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_k}\}$ is a basis for a subspace W of $\mathbb{R}^n$).