Math 224
Quiz 5
Thursday, October 25, 2007

Note: You are allowed to use Maple for computation on this quiz, but you must SHOW ALL WORK to receive credit. Answers for which no work is shown will receive NO CREDIT.

1. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

and the vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v_4} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{v_5} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. List the vectors that are eigenvectors of A, and for the ones that are eigenvectors of A, find the corresponding eigenvalue.

2. Find the characteristic polynomial, the real eigenvalues, and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}.$$

3. Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be a linear transformation, and let λ be a scalar. Prove that $\{\mathbf{v} \in \mathbf{R}^n \mid T(\mathbf{v}) = \lambda\mathbf{v}\}$ is a subspace of $\mathbf{R}^n$.

4. Let A be an $n \times n$ real matrix. Prove that the eigenvalues of A are the same as the eigenvalues of A^T.