Math 224
Quiz 3 Solutions
Thursday, October 11, 2007

1. Find the determinant of
$$A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}.$$

   Solution. Expanding along the second row, we obtain
$$\begin{aligned} \det(A) &= 0 \cdot (-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} + 1 \cdot (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} + 2 \cdot (-1)^{2+3} \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} \\ &= (3-4) - 2 \cdot (12-2) \\ &= -1 - 20 \\ &= -21 \end{aligned}$$

2. Suppose that A is a 3×3 matrix with determinant 2. Find $\det(3A)$.

   Solution. Since A is a 3×3 matrix, A has 3 rows. $3A$ is the matrix obtained by multiplying each entry of A by 3. Thus, if A has row vectors $\mathbf{a_1}$, $\mathbf{a_2}$, and $\mathbf{a_3}$, $3A$ has row vectors $\mathbf{3a_1}$, $\mathbf{3a_2}$, and $\mathbf{3a_3}$. Since multiplying a single row of a matrix A by a scalar r has the effect of multiplying the determinant of A by r, we obtain:
$$\det(3A) = 3 \cdot 3 \cdot 3 \det(A) = 27 \cdot 2 = 54.$$

3. Suppose that A is a 3×3 matrix with row vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, and that $\det(A) = 3$. Find the determinant of the matrix with row vectors $\mathbf{a} + \mathbf{a}$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} + \mathbf{c}$.

   Solution. First, note that $\mathbf{a} + \mathbf{a} = \mathbf{2a}$, so that the first row of the given matrix is obtained my multiplying the first row of A by 2. We know that this has the effect of multiplying the determinant of A by 2. Next, rows 2 and 3 of the given matrix are obtained by adding a scalar multiple of a row of A to *another* row of A, which does not change the determinant. Thus the determinant of the given matrix is
$$2 \cdot \det(A) = 2 \cdot 3 = 6.$$

4. Suppose that A is a square matrix with $\det(A) = 5$. Find $\det(A^T A)$.

   Solution.

$$\begin{aligned} \det(A^T A) &= \det(A^T)\det(A) \\ &= \det(A)\det(A) \\ &= 5 \cdot 5 \\ &= 25. \end{aligned}$$

5. Is the matrix

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 4 & 1 & -2 \\ -5 & 1 & 4 \end{bmatrix}$$

   invertible?

   Solution. $\det(A) \neq 0$, so A is invertible.