Math 224
Quiz 2
Thursday, September 13, 2007

1. Reduce the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 2 \\ 3 & -1 & 6 \end{bmatrix}$ to row-echelon form.

Solution. $\begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 2 \\ 3 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 0 \\ 0 & -10 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. Find all solutions of the given linear system, using the Gauss method with back substitution.

$$\begin{aligned} 2x - y &= 8 \\ 6x - 5y &= 32 \end{aligned}$$

Solution. First we form the augmented matrix $[A|\mathbf{b}] = \left[\begin{array}{cc|c} 2 & -1 & 8 \\ 6 & -5 & 32 \end{array}\right]$. Row reducing, we obtain $[A|\mathbf{b}] \sim \left[\begin{array}{cc|c} 2 & -1 & 8 \\ 0 & -2 & 8 \end{array}\right]$. Thus we conclude $-2y = 8$, so $\boxed{y = -4}$ and $2x - y = 8$, so $2x + 4 = 8$, so $\boxed{x = 2}$.

3. Determine whether the vector $\mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$ is in the span of the vectors $\mathbf{v_1} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}$.

Solution. We need to determine whether or not the system $A\mathbf{x} = \mathbf{b}$ is consistent, where A is the 3×3 matrix consisting of the column vectors $\mathbf{v_i}$. Thus we form the augmented matrix $[A|\mathbf{b}] = \left[\begin{array}{ccc|c} 0 & 1 & -3 & 3 \\ 2 & 4 & -1 & 5 \\ 4 & -2 & 5 & 3 \end{array}\right]$. Next, we row reduce to obtain $[A|\mathbf{b}] \sim \left[\begin{array}{ccc|c} 2 & 4 & -1 & 5 \\ 0 & 1 & -3 & 3 \\ 0 & -10 & 7 & -7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 4 & -1 & 5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & -23 & 23 \end{array}\right]$. Since the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, we conclude that $\boxed{\mathbf{b} \text{ is in the span of the vectors } \mathbf{v_1}, \mathbf{v_2}, \text{ and } \mathbf{v_3}}$.

4. Let $A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$. If possible, find a matrix C such that $AC = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$.

Solution. We have $AC = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$, and since we know A^{-1} exists, multiplication on the left by A^{-1} implies $A^{-1}AC = A^{-1}\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$, or $C = A^{-1}\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$. Performing the matrix multiplication, we obtain $\boxed{C = \begin{bmatrix} 5 & 5 \\ 4 & 4 \\ 12 & 11 \end{bmatrix}}$.