Math 224
Quiz 1 Solutions
Thursday, September 6, 2007

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1. Find all scalars c such that the vector $[c^2, -4]$ is parallel to the vector $[1, -2]$.

   Solution. For $[c^2, -4]$ to be parallel to $[1, -2]$, we need a scalar (real number) r such that $r[c^2, -4] = [1, -2]$. This implies $[rc^2, -4r] = [1, -2]$. Equating the second components, we obtain $-4r = -2$, or $r = 1/2$. Thus $\frac{1}{2}c^2 = 1$, so $c^2 = 2$. Thus $\boxed{c = \pm\sqrt{2}}$.

2. Classify the vectors $[2, 1, 4, -1]$ and $[0, 1, 2, 4]$ as parallel, perpendicular, or neither.

   Solution. First, recall that two vectors are perpendicular if and only if their dot product is zero:

   $$[2, 1, 4, -1] \cdot [0, 1, 2, 4] = 2 \cdot 0 + 1 \cdot 1 + 4 \cdot 2 + 1 \cdot 4 = 0 + 1 + 8 + -4 = 5 \neq 0.$$

   So the vectors are $\boxed{\text{not perpendicular}}$.

   For the vectors to be parallel, we need a scalar r such that $r[2, 1, 4, -1] = [0, 1, 2, 4]$, which is equivalent to $[2r, r, 4r, -r] = [0, 1, 2, 4]$. Equating the first components we obtain $r = 0$, and equating the second components we obtain $r = 1$. Thus no such r exists, so we conclude that the vectors are $\boxed{\text{not parallel}}$.

3. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & 0 \end{bmatrix}$. Let $B = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$. Find AB and BA, if defined.

   Solution. A is a 2×3 matrix and B is a 3×1 matrix. Thus AB is defined and is a 2×1 matrix and $\boxed{BA \text{ is not defined}}$ since the number of columns of B is not equal to the number of rows of A $(1 \neq 2)$.

   $$AB = \begin{bmatrix} 1 \cdot 4 + 0 \cdot 2 + -1 \cdot -2 \\ -2 \cdot 4 + 2 \cdot 1 + 0 \cdot -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

4. If B is an $m \times n$ matrix and if $B = A^T$, find the size of

   Solution.

   (a) A: Since A^T is an $m \times n$ matrix, A is an $n \times m$ matrix.

   (b) AA^T: Since A is an $n \times m$ matrix and A^T is an $m \times n$ matrix, AA^T is an $n \times n$ matrix.

   (c) A^TA: Since A^T is an $m \times n$ matrix and A is an $n \times m$ matrix, A^TA is an $m \times m$ matrix.