Math 224
Properties of Orthogonal Matrices
Answer Key

I have indicated whether each of the statements is True or False. Please see me (or email me) if you have any trouble determining why those that are true are true, or if you have trouble finding a counterexample for those that are false.

1. The identity matrix is orthogonal. **True.**

2. The matrix
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
   is orthogonal. **True.**

3. The matrix
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$$
   where θ is any angle, is orthogonal. **True.**

4. Every diagonal matrix is orthogonal. **False.**

5. If A is an $n \times n$ orthogonal matrix, and $\mathbf{x}$ and $\mathbf{y}$ are any column vectors in $\mathbf{R}^n$, then $(A\mathbf{x}) \cdot (A\mathbf{y}) = \mathbf{x} \cdot \mathbf{y}$. **True.**

6. If A is an $n \times n$ orthogonal matrix, and $\mathbf{x}$ is any column vector in $\mathbf{R}^n$, then $||A\mathbf{x}|| = ||\mathbf{x}||$. It may be useful to remember that the command in Maple for finding $||\mathbf{v}||$ is norm(v,2). **True.**

7. If A is an $n \times n$ orthogonal matrix, and $\mathbf{x}$ and $\mathbf{y}$ are any non-zero column vectors in $\mathbf{R}^n$, then the angle between $\mathbf{x}$ and $\mathbf{y}$ is equal to the angle between $A\mathbf{x}$ and $A\mathbf{y}$. It may be useful to remember that the command in Maple for finding the angle between two vectors $\mathbf{v}$ and $\mathbf{w}$ is angle(v,w). **True.**

8. An orthogonal matrix must be symmetric. **False.**

9. The product of two orthogonal matrices is also orthogonal. **True.**

10. The norm of the first column of an orthogonal matrix must be 1. **True.**

11. The norm of the first row of an orthogonal matrix must be 1. **True.**

12. The vectors formed by the first and last rows of an orthogonal matrix must be orthogonal. **True.**

13. The vectors formed by the first row and the second column of an orthogonal matrix must be orthogonal. **False.**

14. The determinant of an orthogonal matrix is always 1. **False.**

15. Every entry of an orthogonal matrix must be between 0 and 1. **False.**

16. The eigenvalues of an orthogonal matrix are always ± 1. **False.**

17. If the eigenvalues of an orthogonal matrix are all real, then the eigenvalues are always ± 1. **True.**

18. In any column of an orthogonal matrix, at most one entry can be equal to 1. **True.**

19. In any column of an orthogonal matrix, at most one entry can be equal to 0. **False.**

20. If A is an $n \times n$ symmetric orthogonal matrix, then $A^2 = I$. **True.**

21. If A is an $n \times n$ symmetric matrix such that $A^2 = I$, then A is orthogonal. **True.**

22. The nullspace of any orthogonal matrix is $\{\mathbf{0}\}$. **True.**

23. If A is a 2×2 matrix with determinant 1, then A is an orthogonal matrix. **False.**