Math 224
Homework 6 Solutions

Section 4.4

4.4 #2

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}.$$

Using Maple, we compute $\sqrt{\det A^T A} = \sqrt{80} = 4\sqrt{5}$. So the volume of the 2-box in $\mathbf{R}^5$ determined by the given vectors is $\boxed{4\sqrt{5}}$.

4.4 #8

$$A = \begin{bmatrix} -1 & 3 & 4 \\ 4 & -2 & 0] \\ 7 & -1 & 2 \end{bmatrix}.$$

Using Maple, we compute $\det A = 20$, so the volume of the 3-box in $\mathbf{R}^3$ determined by the given vectors is $\boxed{20}$.

4.4 #12 The volume of the the 3-box determined by the vectors

$$\begin{aligned} \mathbf{a_1} &= (-1,2,4) - (1,0,3) = [-2,2,1] \\ \mathbf{a_2} &= (3,-1,2) - (1,0,3) = [2,-1,-1] \\ \mathbf{a_3} &= (2,0,-1) - (1,0,3) = [1,0,-4] \end{aligned}$$

is the determinant of the matrix with the $\mathbf{a_j}$ as column vectors. Thus we construct the matrix

$$A = \begin{bmatrix} -2 & 2 & 1 \\ 2 & -1 & 0 \\ 1 & -1 & -4 \end{bmatrix}.$$

Using Maple, we compute $\det(A) = 7$. The tetrahedron has the same altitude but base only half the area of the base of the parallelogram. Thus the volume of the tetrahedron is (1/2)[(1/3)(altitude)(area of base)]= $\boxed{7/6}$.

4.4 #14 The volume of an n-box with two edges that coincide is 0.

4.4 #18 The standard matrix representation of T is given by

$$A = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}.$$

Using Maple, we compute $\det(A) = 16$. Since the area of the square in $\mathbf{R}^2$ is 1, the area of the image of the square under T is $\boxed{16 \cdot 1 = 16}$.

4.4 #26 The standard matrix representation of T is given by

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Using Maple, we compute $\sqrt{\det(A^T A)} = \sqrt{3}$, and since the area of the square in $\mathbf{R}^2$ is 1, the area of the image of the square under T is $\boxed{\sqrt{3} \cdot 1 = \sqrt{3}}$.

4.4 #32 The standard matrix representation of T is given by

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & -1 \\ 2 & 1 \end{bmatrix}.$$

Using Maple, we compute $\sqrt{\det(A^T A)} = \sqrt{17}$, and since the area of the disk in $\mathbf{R}^2$ is 9π, the area of the image of the disk under T is $\boxed{\sqrt{17} \cdot 9\pi = 9\pi\sqrt{17}}$.

Section 5.1

5.1 #45 See the Maple file fibonacci.mw posted on P:/Class/Math/Paquin/Math224. As in class, let $\mathbf{x_k} = \begin{bmatrix} F_k \\ F_{k-1} \end{bmatrix}$, and let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Then

$$\mathbf{x_k} = A\mathbf{x_{k-1}} = A^{k-1}\mathbf{x_1},$$

where $\mathbf{x_1} = \begin{bmatrix} F_1 \\ F_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(a) $A^7\mathbf{x_1} = \begin{bmatrix} 21 \\ 13 \end{bmatrix}$, so $\boxed{F_8 = 21}$.

(b) $A^29\mathbf{x_1} = \begin{bmatrix} 832040 \\ 514229 \end{bmatrix}$, so $\boxed{F_{30} = 832040}$.

(c) $A^{49}\mathbf{x_1} = \begin{bmatrix} 12586269025 \\ 7778742049 \end{bmatrix}$, so $\boxed{F_{49} = 12586269025}$.

(d) $A^{76}\mathbf{x_1} = \begin{bmatrix} 5527939700884757 \\ 3416454622906707 \end{bmatrix}$, so $\boxed{F_{77} = 5527939700884757}$.

(e) $A^{149}\mathbf{x_1} = \begin{bmatrix} 9969216677189303386214405760200 \\ 6161314747715278029583501626149 \end{bmatrix}$, so $\boxed{F_{150} = 9969216677189303386214405760200}$.

5.1 #46 (a) $a_0 = 0$, $a_1 = 1$, $a_2 = 2$, $a_3 = 5$, $a_4 = 12$, $a_5 = 29$, $a_6 = 70$, $a_7 = 169$, $a_8 = 408$

(b) $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$. As in the Fibonacci sequence example in class, let $\mathbf{x_k} = \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix}$. Then

$$\mathbf{x_k} = A\mathbf{x_{k-1}} = A^{k-1}\mathbf{x_1}.$$

(c) $A^{29}\mathbf{x_1} = \begin{bmatrix} 107578520350 \\ 44560482149 \end{bmatrix}$, so $\boxed{a_{30} = 107578520350}$.