Math 224
Homework 3 Solutions

Section 1.6

1.6 #4: Let $S = \{[x, y] \mid x, y \geq 0\}$. Since $[1, 1]$ is in S, but $-1 \cdot [1, 1] = [-1, 1]$ is *not* in S, we see that S is not closed under scalar multiplication, so $\boxed{S \text{ is not a subspace of } \mathbf{R}^2}$.

1.6 #8: Let $S = \{[2x, x+y, y]\}$. Let $\mathbf{v} = [2a, a+b, b]$ and $\mathbf{w} = [2c, c+d, d]$ be two vectors in S. Then $\mathbf{v}+\mathbf{w} = [2a+2c, a+b+c+d, b+d] = [2(a+c), (a+c)+(b+d), b+d]$, which does have the form $[2x, x+y, y]$. Also, $r\mathbf{v} = r[2a, a+b, b] = [2ra, ra+rb, rb]$ is in S. Thus we conclude that $\boxed{S \text{ is a subspace of } \mathbf{R}^3}$.

1.6#10: $S = \{[x_1, 0, x_3, \ldots, x_n]\}$ $\boxed{\text{is a subspace of } \mathbf{R}^3}$ since S is closed under addition and scalar multiplication.

1.6#12: The plane $ax + by + cz = 0$ can be viewed as the solution set of a homogeneous linear system consisting of a single equation. Thus, by Theorem 1.13, the plane $ax + by + cz = 0$ $\boxed{\text{is a subspace of } \mathbf{R}^3}$.

1.6#18: The augmented matrix $[A|\mathbf{0}] = \left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 3 & 0 \end{array}\right]$. Row reducing in Maple, we obtain $[A|\mathbf{0}] = \begin{array}{cccc|c} 1 & 0 & 0 & 3/5 & 0 \\ 0 & 1 & 0 & 4/5 & 0 \\ 0 & 0 & 1 & -4/5 & 0 \end{array}]$. The last column does not contain a pivot, so x_4 is a free variable. Set $x_4 = r$. This implies $\mathbf{x} = r \begin{bmatrix} -3/5 \\ -4/5 \\ 4/5 \\ 1 \end{bmatrix}$.

Thus $\boxed{\{[-3/5, -4/5, 4/5, 1]\} \text{ is a basis for the solution set}}$.

1.6#26: The matrix $\begin{bmatrix} 2 & 2 & 3 & 5 \\ 1 & -3 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}$ is row equivalent to the identity, so it is invertible, so the set of vectors $\boxed{\text{is a basis for the subspace of } \mathbf{R}^4 \text{ that the vectors span}}$.

1.6#32: We row reduce the augmented matrix $[A|\mathbf{0}]$ to $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. Thus x_3 and

x_4 are free variables, and the solution space is given by $\begin{bmatrix} -2s-r \\ -s-2r \\ s \\ r \end{bmatrix}$. Thus $\boxed{\{[-2,-1,1,0],[-1,-2,0,1]\} \text{ is a basis for the nullspace}}$.

Section 2.1

2.1 #2: If two of the vectors are dependent, then we don't have anything to prove. If two of the vectors are independent, then they span all of $\mathbf{R}^2$, so the third vector must be a linear combination of the first two, so the three vectors must be dependent.

2.1 #4: An independent set of two vectors in $\mathbf{R}^3$ spans a plane.

2.1 #10: We row reduce the matrix $A = \begin{bmatrix} -2 & 3 & 1 & -1 \\ 3 & -1 & 2 & 5 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ to obtain $rref(A) = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The first and second columns contain a pivot, so $\boxed{\{[-2,3,1],[3,-1,2]\} \text{ is a basis}}$.

2.1 #12: $rref(A) = \begin{bmatrix} 1 & 0 & 1/11 \\ 0 & 1 & 3/11 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The first and second columns contain a pivot, so $\boxed{\{[2,5,1,6],[3,2,7,-2]\} \text{ is a basis}}$.

2.1 #22: We row reduce the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -4 & -11 & -3 \\ 3 & 2 & -4 \end{bmatrix}$ to obtain $rref(A) = I$, the 3×3 identity matrix, so the given set of vectors is $\boxed{\text{independent}}$.

2.1 #26: First, we form the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. Row reducing, we obtain

$rref(A) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$. The first, second, and third columns contain pivots, so a basis for $\mathbf{R}^3$ is $\boxed{\{[1,2,1],[1,0,0],[0,1,0]\}}$.

2.1 #32: We row reduce the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & s & 3 \\ 1 & 3 & 1 \end{bmatrix}$ to obtain $rref(A) = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3+s \end{bmatrix}$.
We see that the third column will not have a pivot if $s = -3$, so $\boxed{\text{the vectors are independent for all scalars } s \neq -3}$.