Math 224
Homework 12 Solutions

Section 3.4

3.4 #22: (a)

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Let $p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$. Using coordinate vectors relative to B, the equation $T(p(x)) = x^3 - 3x^2 + 4x$ becomes

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & a_1 \\ 0 & 2 & 0 & 0 & a_2 \\ 0 & 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 0 & a_0 \end{array}\right] = \begin{bmatrix} 1 \\ -3 \\ 4 \\ 0 \end{bmatrix}.$$

The solution is

$$a_1 = 1/3, a_2 = -3/2, a_3 = 4, a_4 = c,$$

where c is any scalar. Thus

$$p(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x + c,$$

where c is any scalar.

(c) The matrix representation A_1 of T_1 relative to B, B'' is

$$A_1 = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The matrix representation A_2 of T_2 relative to B'', B is

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then

$$A_2A_1 = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A.$$

(d)
$$A_1A_2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$
Thus $T_3(a_2x^2 + a_1x + a_0) = 3a_2x^2 + 2a_1x + a_0$, so $T_3 = T_1 \circ T_2$.

3.4 #24: (a)
$$A = \begin{bmatrix} 24 & 0 & 0 & 0 \\ 24 & 8 & 0 & 0 \\ 6 & 4 & 2 & 0 \end{bmatrix}.$$

(b) $T(4x^3 - 5x^2 + 4x - 7) = 96x^2 + 56x + 12$.

3.4 #27:
$$A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 81 \end{bmatrix}.$$

3.4 #31:
$$A = \begin{bmatrix} -3 & -3 \\ 4 & -4 \end{bmatrix}.$$

3.4 #32: $T(a\sin 2x + b\cos 2x) = -2b\sin 2x + 2a\cos 2x$.