Math 224
Homework 11 Solutions

Section 3.2

3.2 #2: The set is NOT a subspace of P since it is not closed under vector addition. For example, $p(x) = x^4 + x^3$ and $q(x) = -x^4$ are both in the set, but $p(x) + q(x) = x^3$ is not in the set.

3.2 #3: The set is NOT a subspace of F since it is not closed under vector addition. If $f(0) = g(0) = 1$, then $(f+g)(0) = f(0) + g(0) = 1 + 1 = 2$.

3.2 #4: W is a subspace of F. You should verify that W is non-empty, closed under vector addition, and closed under scalar multiplication.

3.2 #5: S is a subspace of W. You should verify that S is non-empty, closed under vector addition, and closed under scalar multiplication.

3.2 #8: Note that

$$1 = 1(1 + 2x) + (-2)x$$

and

$$x = 0(1 + 2x) + 1(x),$$

so $\text{sp}(1, x)$ is contained in $\text{sp}(1 + 2x, x)$. Next,

$$1 + 2x = 1(1) + 2(x)$$

and

$$x = 0(1) + 1(x),$$

so $\text{sp}(1 + 2x, x)$ is contained in $\text{sp}(1, x)$. Thus we conclude that

$$\text{sp}(1, x) = \text{sp}(1 + 2x, x).$$

3.2 #12: The set of vectors is dependent. Suppose

$$r_1 + r_2(4x + 3) + r_3(3x - 4) + r_4(x^2 + 2) + r_5(x - x^2 = 0.$$

Then

$$(r_4 - r_5)x^2 + (4r_2 + 3r_3 + r_5)x + (r_1 + r_2 - 4r_3 + 2r_4) = 0.$$

Thus we solve the system

$$\begin{aligned} r_4 - r_5 &= 0 \\ 4r_2 + 3r_3 + r_5 &= 0 \\ r_1 + 3r_2 - 4r_3 + 2r_4 &= 0 \end{aligned}$$

We row reduce the augmented matrix

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right]$$

to obtain

$$\left[\begin{array}{ccccc|c} 1 & 0 & -25/4 & 0 & 5/4 & 0 \\ 0 & 1 & 3/4 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right].$$

Since the third and fifth columns do not contain a pivot, r_3 and r_5 are free variables, so we can easily find a non-trivial solution for r_1, r_2, r_3, r_4, r_5. Thus the set is dependent.

3.2 #13: Since $\cos^2 x = 1 + (-1)\sin^2 x$, the set of vectors is dependent.

3.2 #14: Suppose $r_1 \sin x + r_2 \cos x = 0$. Setting $x = 0$, we obtain $r_2 = 0$. Setting $x = \pi/2$, we obtain $r_1 = 0$. Thus the set of vectors is independent.

3.2 #18: Suppose that $r_1 e^{2x} + r_2 e^{3x} + r_3 e^{4x} = 0$. Differentiating twice, we obtain the two additional equations:

$$\begin{aligned} 2r_1 e^{2x} + 3r_2 e^{3x} + 4r_3 e^{4x} &= 0 \\ 4r_1 e^{2x} + 9r_2 e^{3x} + 16r_3 e^{4x} &= 0 \end{aligned}$$

Substituting $x = 0$ in these three equations yields the following homogeneous linear system:

$$\begin{aligned} r_1 + r_2 + r_3 &= 0 \\ 2r_1 + 3r_2 + 4r_3 &= 0 \\ 4r_1 + 9r_2 + 16r_3 &= 0 \end{aligned}$$

Solving this system, we obtain $r_1 = r_2 = r_3 = 0$, so the set of vectors is independent.

3.2 #20: $(x-1)^2 = (x^2+1) + (-2)x$, so the set of vectors is dependent and hence is NOT a basis for P_2.

Section 3.3

3.3#4: The coordinate vector is $[1, 2, -1]$.

3.3#6: The coordinate vector is $[-1, 2, 1, 3]$.

3.3#8: The coordinate vector is $[-4, -2, 1, 5]$.

3.3#12: The coordinate vector is $[4, 3, -5, 4]$.

3.3#21: The polynomial is

$$3(x + x^2) + 1(x - x^2) + 2(1 + x) = 2x^2 + 6x + 2.$$