
Math 224
Thursday, October 4, 2007
Properties of Determinants

Using the `det` command in Maple to conduct experiments on various 3×3 and 4×4 matrices (and/or matrices of larger sizes), make conjectures about the following statements/questions/properties. Once you have made your conjectures, try to think about how to prove them using the definition of the determinant and the theorem on general expansion by minors for general $n \times n$ matrices.

1. The Transpose Property. For any matrix A , how is $\det(A)$ related to $\det(A^T)$?

2. The Row-Interchange Property. If two different rows of a square matrix A are interchanged, what is the determinant of the resulting matrix?

3. The Equal-Rows Property. If two rows of a square matrix A are equal, what is the value of $\det(A)$?

4. The Scalar Multiplication Property. If a single row of a square matrix A is multiplied by a scalar r , what is the determinant of the resulting matrix?

5. The Row-Addition Property. If the product of one row of a square matrix A by a scalar is added to a different row of A , what is the determinant of the resulting matrix?

6. The Multiplicative Property. If A and B are square matrices, how is $\det(AB)$ related to $\det(A)$ and $\det(B)$?

7. Determinant of a Diagonal Matrix. If A is an $n \times n$ diagonal matrix, what is the value of $\det(A)$?

8. Determinant Criterion for Invertibility. Complete the following sentence: A square matrix A is invertible if and only if $\det(A)$

9. If $rref(A)$ contains a row of zeros, what is the value of $\det(A)$?

10. How is the determinant of an $n \times n$ matrix A related to the pivots in the row echelon form of A using only row addition and row interchanges (i.e. no row scaling)?

11. If A is an invertible square matrix, how is $\det(A^{-1})$ related to $\det(A)$?