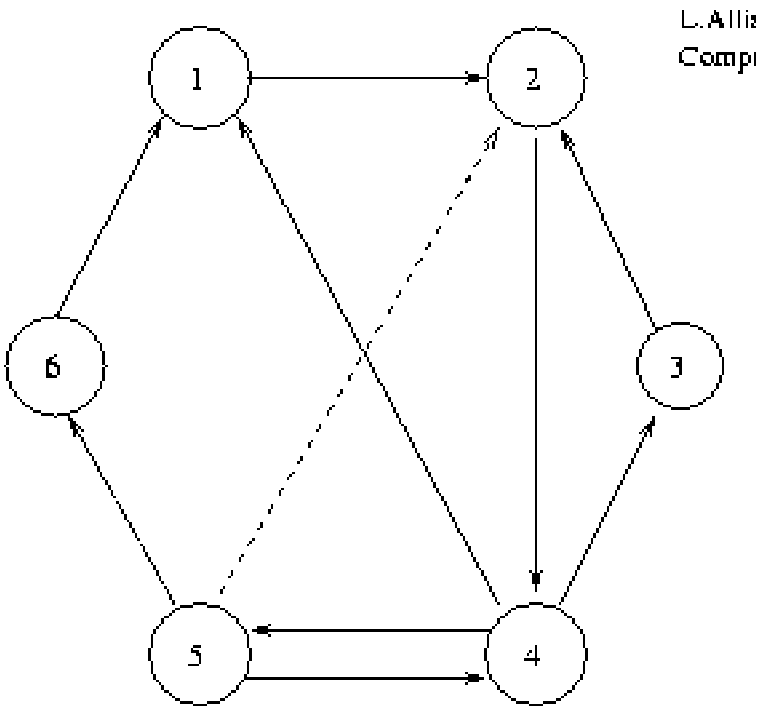
Math 224
Application of Matrix Operations

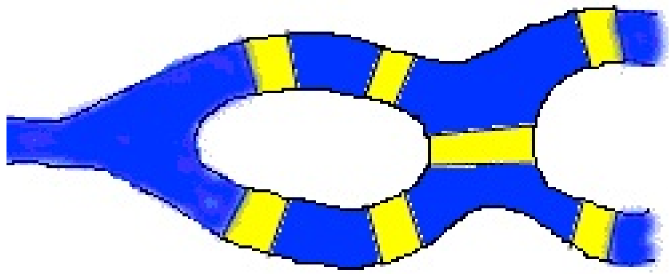
A graph consists of a set of vertices and a set of edges that connect (some of) the vertices. The arrows on the edges distinguish between two-way connections and one-way connections. Such a graph can be described by an $n \times n$ matrix called an adjacency matrix in which 1's and 0's are used to describe connections. Specifically, if the vertices are labeled from 1 to n, then the entry in row i and column j of the adjacency matrix is a 1 if there is a connection *from* vertex i *to* vertex j, and a 0 if there is not.

Construct the adjacency matrix for the graph in the following figure, and let A denote the matrix. Compute A^2, A^3, and A^4, and interpret the results in terms of the graph. Try to generalize the meaning of A^n for positive integers n.



Directed Graph

Note: this problem is related to the Bridges of Konigsberg problem, posed to Leonhard Euler in the mid 1700s by the citizens of the Prussian city of Konigsberg (now Kaliningrad in Russia). The city is cut by the Pregel River, which encloses and island, as shown in the following figure:



The problem was to determine whether it was possible to start at any point on the shore of the river, or on the island, and walk over all of the bridges, once and only once, returning to the starting spot. In 1736 Euler showed that the walk was impossible by analyzing the graph.