Tests for Convergence of Series

1) Use the comparison test to confirm the statements in the following exercises.

1. $\sum_{n=4}^{\infty} \frac{1}{n}$ diverges, so $\sum_{n=4}^{\infty} \frac{1}{n-3}$ diverges.

2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, so $\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ converges.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, so $\sum_{n=1}^{\infty} \frac{e^{-n}}{n^2}$ converges.

2) Use the comparison test to determine whether the series in the following exercises converge.

1. $\sum_{n=1}^{\infty} \frac{1}{3^n+1}$
2. $\sum_{n=1}^{\infty} \frac{1}{n^4+e^n}$
3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$
4. $\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$
5. $\sum_{n=1}^{\infty} \frac{n \sin^2 n}{n^3+1}$
6. $\sum_{n=1}^{\infty} \frac{2^n+1}{n2^n-1}$

3) Use the ratio test to decide if the series in the following exercises converge or diverge.

1. $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$
2. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$
3. $\sum_{n=1}^{\infty} \frac{(2n)!}{n!(n+1)!}$
4. $\sum_{n=1}^{\infty} \frac{1}{r^n n!}, r > 0$
5. $\sum_{n=1}^{\infty} \frac{1}{ne^n}$
6. $\sum_{n=0}^{\infty} \frac{2^n}{n^3+1}$

4) Use the integral test to decide whether the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n}$
4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

5) Use the alternating series test to show that the following series converge.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$
2. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$
3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+2n+1}$

4. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{e^n}$

6) In the following exercises determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1. $\sum \frac{(-1)^n}{2^n}$
2. $\sum \frac{(-1)^n}{2n}$
3. $\sum (-1)^n \left(1 + \frac{1}{n^2}\right)$
4. $\sum \frac{(-1)^n}{n^4+7}$
5. $\sum \frac{(-1)^{n-1}}{n \ln n}$
6. $\sum \frac{(-1)^{n-1} \arctan(1/n)}{n^2}$

7) In the following exercises use the limit comparison test to determine whether the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{5n+1}{3n^2}$, by comparing to $\sum_{n=1}^{\infty} \frac{1}{n}$
2. $\sum_{n=1}^{\infty} \left(\frac{1+n}{3n}\right)^n$, by comparing to $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$
3. $\sum \left(1 - \cos \frac{1}{n}\right)$, by comparing to $\sum 1/n^2$
4. $\sum \frac{1}{n^4-7}$
5. $\sum \frac{n^3-2n^2+n+1}{n^4-2}$
6. $\sum \frac{2^n}{3^n-1}$
7. $\sum \left(\frac{1}{2n-1} - \frac{1}{2n}\right)$
8. $\sum \frac{1}{2\sqrt{n}+\sqrt{n+2}}$

8) Explain why the integral test cannot be used to decide if the following series converge or diverge.

1. $$\sum_{n=1}^{\infty} n^2$$
2. $$\sum_{n=1}^{\infty} e^{-n} \sin n$$

9) Explain why the comparison test cannot be used to decide if the following series converge or diverge.

1. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$
2. $$\sum_{n=1}^{\infty} \sin n$$

10) Explain why the ratio test cannot be used to decide if the following series converge or diverge.

1. $$\sum_{n=1}^{\infty} (-1)^n$$
2. $$\sum_{n=1}^{\infty} \sin n$$

11) Explain why the alternating series test cannot be used to decide if the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}n$

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\left(2-\frac{1}{n}\right)$

12) JAMBALAYA!!! Determine if the following series converge or diverge.

1. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

2. $\sum_{n=1}^{\infty} \frac{n2^n}{3^n}$

3. $\sum_{n=0}^{\infty} e^{-n}$

4. $\sum_{n=1}^{\infty} \frac{1}{n^2} \tan\left(\frac{1}{n}\right)$

5. $\sum_{n=1}^{\infty} \frac{5n+2}{2n^2+3n+7}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{3n-1}}$

7. $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

8. $\sum_{n=2}^{\infty} \frac{3}{\ln n^2}$

9. $\sum_{n=1}^{\infty} \frac{n(n+1)}{\sqrt{n^3+2n^2}}$